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Factors Affecting Teacher Self-efficacy in Elementary Mathematics Instruction

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FACTORS AFFECTING

TEACHER SELF- EFFICACY

IN ELEMENTARY MATHEMATICS INSTRUCTION

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ABSTRACT

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The purpose of this study was to determine the effects of instructional strategies on teacher self-efficacy beliefs about teaching mathematics to more fully understand the relationship between the two groups, and two determine what factors, if any, improve mathematics teaching self- efficacy beliefs of teachers.

The study was conducted in elementary schools using anonymous self- report teacher surveys. Usable data were received from 93 teachers in 47 elementary schools in the Diocese of Rockville Centre, Long Island, New York.

The findings revealed that respondents' perceived ability to provide feedback and clarification, as well as accommodating individual student needs, were the two principal factors which explain the variance in teacher's self- efficacy beliefs. These two factors themselves are influenced by the teachers' understanding mathematical concepts. From the data gathered in this study, we can conclude although teachers may welcome student questions, they do not always feel confident in their ability to answer these questions sufficiently. The data also revealed that overall, teachers lack confidence in their performance in front of superiors, their ability to get students interested in mathematics, as well as their ability to increase student retention. The instructional practices of the respondents were more traditional, and teacher- centric; the data revealed that this was related to underlying beliefs about mathematics instruction as well as the respondents' perceived understanding of mathematical concepts

DEDICATION

I would like to thank my husband, Nick for your love and support. You have encouraged me to follow my dreams since we were both teenagers. Your patience and support has kept me going.

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CHAPTER 1

Introduction

Mathematics and its importance to students' success in STEM careers has been an issue in the educational system, going back to 1983 when the National Commission on Excellence in Education (NCEE) published *A Nation at Risk.* This report, commissioned by Ronald Reagan in 1981 was a "battle cry" against supposed mediocrity in America's educational system (Goldstein, p. 165). After eighteen months of assessing the educational system at the time, the report was completed in April 1983. The introduction to the report stated:

> "Our Nation is at risk. Our once unchallenged preeminence in commerce, industry, science, and technological innovation is being overtaken by competitors throughout the world. This report is concerned with only one of the many causes and dimensions of the problem, but it is the one that undergirds American prosperity, security, and civility" (National Commission on Excellence in Education, (NCEE), p. 1, 1983).

The report critiqued the teachers as well as the curriculum in American public and private schools, recommended an improvement to teacher quality, and made a priority of raising expectations of students through increased rigor in curriculum (Goldstein, NCEE). The findings, found that only about only one third of high school students at the time could solve a mathematics problem requiring several steps. Students lacking skills such as analysis and problem- solving would not be able to keep up with the technological advances predicted at the time. Therefore, specific emphasis was given to the need for increased support for mathematics and science education, which was essential for success in the information age society was entering in 1983 (NCEE, 1983).

Thirty years after the publication of *A Nation at Risk,* the forecast for the future of STEM (Science, Technology Engineering and Mathematics, President's Council of Advisors on Science and Technology (PCAST) 2012, p. 9) education in our nation does not seem to have improved. The President's Council of Advisors on Science and Technology (PCAST) comprised of the nation's leading scientists and engineers issued a report in 2012 for President Obama entitled *Engage to Excel: Producing One Million Additional College Graduates with Degrees in Science, Technology, Engineering and Mathematics.* This report stated that our country will need approximately 1 million more STEM professionals than the current rate to maintain our standing in science and technology. This means that our educational system will need to increase students with STEM related undergraduate degrees by about 34% over the present rate (President's Council of Advisors on Science and Technology (PCAST 2012, p. i).

Retention of students in STEM majors is also addressed. Fewer than 40 % of students who enter college for STEM majors, and many of the students who abandon these majors often have difficulty with math required for success in STEM courses (PCAST, p. 5, 2). In fact, "among students who take the ACT entrance examination for college, just 43% achieve the ACT College Readiness Benchmark in math" (PCAST, 2012, p. 27).

PCAST recommends that colleges and universities "catalyze widespread empirically validated teaching practices" (PCAST, 2012, p. iii). The report states that reducing the math preparation gap is one of the most urgent challenges in preparing the workforce for the 21st century. "Closing this gap will require coordinated action on many fronts starting in the earliest grades" (PCAST 2012, p. vi).

As stated in a 2010 report by The Georgetown University Center on Education and the Workforce, entitled *Help Wanted: Projections of Jobs and Education Requirements Through 2018,* "education is a gateway to further training and greater earning potential" (Carnevale, Smith & Strohl, 2010, p. 1). It states that postsecondary education is the key to jobs which would have the most employer- provided training and therefore job advancement. Access to technology on the job is important to worker's earning potential… "even high school dropouts who use technology at work earn about 15 percent more than those who do not," (Carnevale, et al., p. 2). This report contained data which shows that the middle class in America is dispersing into the upwardly mobile 'college- haves' and downwardly mobile 'college have-nots'. Those who are not college graduates are on the 'down- escalator' of social mobility, falling out of the 'disappearing middle class' (Carnevale et al., p. 3). In 1970, 39% of high school dropouts were in the lower income class, whereas in 2007, 59% of high school dropouts were in the lower middle class. This trend was similar for the following educational distributions: high school dropouts, high school graduates, some college/Associate's degree. For those with Bachelor's degree and Graduate degree the trend was the opposite. In 1970, 47% of those with Bachelor's degrees were in the middle income and only 37% were in the upper income category. In 2007, 38% of those with Bachelor's degrees were in the middle-income category and 48% were in the upper income category (Carnevale, et al., p. 3).

The PCAST report of 2012 claimed that the need for STEM knowledge extends to all Americans, as it will play an increasing role in their lives. "A democratic society in which large numbers of people are unfamiliar or uncomfortable with scientific and technological advances faces a great economic disadvantage in globalized competition," (PCAST, 2012, p. 1).

Undergraduate students who will later become K-12 teachers must be more knowledgeable in mathematics, and be able to inspire their students to be curious, active mathematical learners (PCAST, 2010, p. iii). Teachers are the most crucial factor in the educational system, but there are gaps in our knowledge about how to produce and retain them. Two factors seem to be of utmost importance: deep content area (mathematical) understanding, and mastery of pedagogy. Although some researchers and educators dispute the need for an increase in the number of STEM graduates, and the notion that our country may suffer in the future, (Berliner & Glass, 2014), the need for improvement in mathematics education is an issue of importance for all students - not just those who are interested in a future in the STEM fields.

In the decades between President Reagan and President Obama, the federal government sought to improve the crisis in our schools. In 1989, the National Research Council, whose purpose is to further information about science and technology, (as well as advising the federal government), published a report entitled, *Everybody Counts*. This report was written in response to the "urgent need to revitalize mathematics and science education", (NRC, 1989, iii). It examined mathematics education from Kindergarten through graduate school, identifying problems as well as outlining strategies for the future through the year 2000 (NRC, 1989, iii). "Wake up, America! Your children are at risk!" (p.1). That risk was identified as the math- preparation gap.

The gravity of the situation cannot be understated. Understanding of mathematics permeates our lives from the job to the voting booth; it is necessary for completing everyday household tasks as well as understanding health and environmental issues, political debates, and even solutions to social problems facing our nation (NRC, 1989). Poor quantitative literacy can have serious social consequences in our society. "Because mathematics holds the key to leadership in our information-based society, the widening gap between those who are mathematically literate and those who are not coincides with racial and economic categories…. Unless corrected, innumeracy and illiteracy will drive America apart," (NRC, 1989, p. 14).

In the technologically advancing society, the report stated that numeracy, or quantitative literacy, is as important as verbal literacy. Quantitative literacy requires not just an understanding of math facts, but a deeper understanding of mathematical concepts (NRC, 1989, p. 8).

The report stated that negative mathematics attitudes were common among students and their families, and found that peer pressure often made it socially unacceptable to excel in mathematics (p.10). The report recommended instructional practices that would engage students, build their confidence, and involve them in their own learning, constructing their own understanding of mathematics, thereby making mathematics both exciting and relevant (p. 82). The report recommended that the nation build a consensus as to what students ought to learn in mathematics and praised the standards published in 1989 by the National Council of Teachers of Mathematics, (NCTM) which provided specific objectives (p. 89, 91) to improve the "national underachieving curriculum," (p. 45). These standards recommended instruction that was conceptually based rather than computationally based, which would foster mathematical insight, reasoning and problem solving.

In the past two decades, legislation, such as the No Child Left Behind(NCLB) Act of 2001, the Race to the Top Initiative of 2009, and more recently, Every Child Succeeds Act of 2016 focused on improving schools by using standardized testing and by recruiting highly qualified and effective teachers in the STEM fields (House Resolution 1(2001); The White House, 2009: Senate Resolution 1177, 2016).

The recent publication of the Evaluation of the Achievement Levels for Mathematics and Reading on the National Assessment of Educational Progress, (National Academies of Sciences, Engineering, and Medicine (NASEM), 2017), reported that "overall, for the three grades tested, fourth, eighth and twelfth, only a small percentage, (18%, 25%, and 16% respectively) reached the proficient level, and fewer than 5% scored in the advanced category" (p. 85). The test and mathematics achievement levels in this report, commonly known as the "Nation's Report Card" (p. 15), were set by a panel of representatives from the National Committee of Teachers of Mathematics (NTCM) (p. 66), and were intended to represent "the subject matter and skills that the nation wanted the students to know and be able to do" (p. 77). The test, administered throughout the United States, attempts to ensure an accurate picture of student performance by administering the test to students who represent the nation as a whole ("'NAEP sample design'", 2017).

There has been much research related to the need to bridge the mathematics preparation gap, the effort to improve mathematics education, and the phenomenon of negative mathematical attitudes among students and teachers alike (Allen, 2003; Cochran-Smith, 2001: Gujarati, 2010; Guskey, 2000; Hayes, 2016; Kirschner, Sweller, & Clark, 2006; Krupa, 2011; Ojose, 2008; Powell & Kalina, 2009: Smith, 2004; Sparrow & Frid, 2009; Ulrich, Tillema, Hackenberg, & Norton, 2014).

Although standards and testing have been implemented over the past two decades, rather than having built mathematical proficiency and confidence in students, math is considered, in the words of the authors, Boaler and Dweck (2016), a subject that is either dead and irrelevant or considered a “scary four-letter word” by teachers and students alike (Beilock & Willingham, 2014, p. 29). Dr. Jo Boaler of Stanford University presented a concept of the “myth of the mathematically gifted child” (Boaler & Dweck, 2016, p. 94) in which she describes a public perception that mathematics is considered a harder subject than others and to succeed requires innate ability, therefore de-emphasizing learning strategies (Boaler & Dweck, 2016).

While some authors and researchers such as Melinda Ann Smith (2004)) have questioned whether all people need the same mathematics requirements to graduate from high school, (p. 142), even she admitted that the average citizen must have more knowledge of math and science to thrive in the world today than in the past (p. 151). As Dr. Eugene Geist (2010) noted, “creating a country of ‘mathophobes’ does not bode well for us in the uncertain global economy of the future,” (p. 29). Citizens of the twenty- first century need to be able to integrate technology into their work and daily lives. This integration means not only using technology, but being able to analyze the information which is so easily accessed. This includes critical thinking and understanding of how data is compiled and used (Framework for 21st Century Learning. (n.d.) Retrieved June 3, 2017, from http://www.p21.org/about-us/p21-framework).

Twenty-five years after the report, *Everybody Counts* urged educators to create learning environments which would make mathematics exciting and relevant, it was noted by cognitive psychologists Beilock and Willingham (2014) that it is still socially acceptable to dislike or to be 'bad at math'- something that people would refrain from admitting about reading ability (p. 29). Boaler and Dweck (2016) described this dislike of math as being caused by negative experiences in school, leading students to develop negative beliefs which may permeate the rest of their lives. "If school math classrooms presented the true nature of math, we would not have this nationwide dislike of math and widespread math underachievement" (Boaler & Dweck, 2016, p. 23).

In an article for the American Scholar Magazine entitled, *School Reform Fails the Test.* Mike Rose, a research professor at the Graduate School of Education and Information at UCLA, and author of 12 books lamented, "How can we make our schools better when we've made our teachers the problem and not the solution?" (Rose, 2015, p. 1).

Studies into the relationship between teacher's dislike of mathematics and the effect these negative attitudes have on their students in the both the US and in countries around the world (Beilock, Gunderson, Ramirez, & Levine, 2010; Bekdemir, 2010; Boaler & Dweck, 2016; Klinger, 2009; Geist, 2010, 2015; Maclellan, 2012; Phelps, 2010; Wyatt, 2008). Research has shown that mathematics teaching anxiety and mathematics anxiety are related (Haciomeroglu, 2015; Peker & Ertekin; 2011) and that mathematics teaching anxiety is influenced by mathematics self- efficacy beliefs of teachers (Peker, 2016). Teachers carry their perceptions and attitudes into the classroom. Teachers with profoundly negative beliefs may pass these attitudes onto their students. This is an issue worth addressing. As stated by Maclellan, (2012), "impoverished teaching cannot, by default be legitimized by teachers' dislike, fear or ignorance of number and quantification" (Maclellan, 2012, p. 11).

Teachers with low self-efficacy beliefs are prone to devaluing the subject area, as well as passive or cynical teacher behavior. Teachers with over exaggerated self- efficacy beliefs are prone to devaluing learning and growing as teachers (Wyatt, 2014, p. 120). Both may lead to cognitive dissonance and leave teachers less open to the "doubt and reflection which would help them learn," (p. 120), as well as leading them to engage in task avoidance. In other words, if a teacher's beliefs are not aligned with ability, to maintain stability in their conceptions about themselves, they will avoid teaching mathematics or devalue the topic. In addition, learning goals of students are influenced by two main factors: social support and most strongly, classroom environment. If a teacher's classroom dialogue is aimed at higher order thinking, students' learning goals are more likely aimed at mastery (Phelps, 2010). A positive classroom environment would be one that does not make a learning goal of passing a state assessment exam. Students' strengths would be used to challenge them with more complex mathematical tasks. Students' weaknesses would be addressed so that students can continue to make progress (Smith, 2004, p 160).

Problem Statement

It has been established that teachers with higher self- efficacy beliefs will demonstrate more persistence with students, and be more open to innovative ideas, and put more time into planning for mathematics instruction (Nurlu, 2015). In contrast, teacher efficacy beliefs which are misaligned with practical knowledge of instructional strategies and or skills may cause a disconnect with the intended outcome of instruction (Wyatt, 2014). The result may be a state of discomfort known as cognitive dissonance

(Festinger, 1962; Harmon- Jones, 2012) leading to task avoidance and anxiety about teaching math. Using "social constructivist" teaching strategies which require active student engagement - "hands on" as well as "minds on," (Warwick, 2008) has been shown to improve teacher self- efficacy beliefs and lower their anxiety (Hayes, 2016; Phelps, 2009; Warwick, 2008; Weber, 2006; Wyatt, 2014). Yet, as Phelps noted in 2009,

> "The reasons for a relationship between social constructivist courses and increased mathematics self-efficacy are not well understood. It may be that classrooms designed around social constructivist principles have a decreased emphasis on competition, leading students to focus on their own understanding, draw fewer social comparisons, and therefore, raise their beliefs about their own abilities. More research is needed in understanding classroom environment as a source of mathematics self-efficacy." (Phelps, 2009 p. 48).

Statement of Purpose

To support educational leadership and promote student learning, the researcher will attempt to find out what role several specific factors of social constructivist teaching strategies: inquiry, classroom discourse, collaboration and their associated outcome, reflection in action contribute to self- efficacy of elementary mathematics teachers.

Research Questions

1. What are self- efficacy beliefs of teachers of elementary mathematics?
2. What instructional strategies characterize those of elementary mathematics teachers?
3. What are teachers' beliefs about student learning and mathematics instruction?
4. What is the relationship between reflection in action, classroom discourse, and teacher self- efficacy beliefs in mathematics instruction?

Overview

This correlational study will investigate the nature of the relationship between mathematics self- efficacy of teachers and the use of collaboration, inquiry and discourse in the classroom.

Hypothesis

Null Hypothesis: There will be no relationship between the use of inquiry, collaboration, classroom discourse, reflection in action and mathematics self- efficacy beliefs of teachers.

Alternative Hypothesis: There will be correlation between the use of inquiry, collaboration, classroom discourse, reflection in action and mathematics self- efficacy beliefs of teachers.

Research Objective

The study is an integrative approach. It will be a psycho- educational appraisal of a target area; feeling states will be examined as well as behavioral interventions.

Rationale and Significance of the Study

Although, as previously stated, dislike of mathematics as well as teaching mathematics is a problem studied worldwide, the study will be limited to the mathematics self- efficacy of elementary school teachers on Long Island.

One goal of the 1989 report, *Everybody Counts* was to build student confidence and make mathematics exciting and relevant (NRC, 1989). The significance of this present study was summed up by the NRC in 1989:

> "There is little we do in America that is more important than teaching. Effective teaching of mathematics requires appropriate pedagogical and mathematical foundations, but thrives only in an environment of trust which encourages leadership and innovation," (NRC, 1989 p. 57).

More than 25 years later, mathematics is still looked on with trepidation and fear by both teachers and their students and (Beilock & Willingham, 2014). Confidence in mathematics makes a positive difference in the curriculum and instructional choices teachers make (Geist, 2015). Teachers' mathematical self- efficacy beliefs are comprised of several sub- factors, among them are the belief that they can motivate students to take on responsibility, and their belief in providing effective teaching.

There are still obstacles to evolving mathematics instruction. It requires a shift from traditional, procedural, skill- based instruction to more innovative strategies. The teachers using more innovative strategies report more confidence (Hayes, 2016; Sparrow & Frid, 2009; Warwick, 2008) yet the reason for this is not clear. This study will examine the relationship between student- teacher feedback and teacher efficacy and motivation. A key factor seems to be that the use of inquiry, collaboration and discourse provide opportunities for the teacher to reflect upon student understanding of concepts as students acquire quantitative literacy.

Conceptual Framework:

Using the theories of Vygotsky, Boaler, Dweck, and Bandura, the researcher will explore the connections between the use of social constructivist instruction in mathematics and teachers' mathematics self- efficacy.

Definitions of terms:

- *Cognitive dissonance theory*- when individual holds two or more elements of knowledge that are relevant to each other but inconsistent with one another, a state of discomfort or dissonance is created (Festinger, 1962; Harmon- Jones, 2012).

- *Mathematical beliefs:* the perspective and experiences regarding mathematics which teachers have acquired and bring into their teacher education programs and classrooms (Haciomeroglu, 2015; Hughes, 2016).
- *Mathematics teaching anxiety*: a feeling of tension and fear that takes place during teaching mathematical concepts (Haciomeroglu, 2015; Hayes, 2016; Hembree, 1990, Maloney, Schaeffer, & Beilock, 2013; Suinn & Winston, 2003; Weber, 2006).
- *Reflection in Action (also known as professional noticing)*- attending to student behavior, interpreting student learning, and responding based on student understanding (Maclellan, 2012, p. 11; Sch n, 1983, p. 68).
- *Teacher mathematic self- efficacy:* a personal cognition, a teacher's individual beliefs in their capabilities to perform mathematical teaching tasks at a specified level of quality in a specified level of quality in a specified situation, based on the self- perception of one's mathematical ability (Bandura, 1977; Dellinger, Bobbett, Olivier & Ellett; Griggs, Rimm- Kaufman, Merritt, & Patton; Usher & Pajares, 2009; Warwick, 2008).
- *Social Development Theory:* -consciousness and cognition are the result of social interaction (Vygotsky, Cole, John- Steiner, Scribner, & Souberman, 1978).
- *Constructivism:* "learning is a deeply personal activity involving the examining of beliefs and prior knowledge in the light of learning experiences and the teaching context," (Wyatt, p. 124).
- *Zone of proximal development:* "the distance between the actual development level as determined by independent problem- solving and the level of potential

development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, et al., 1978, p. 86).

- *Social Constructivist Theory of Learning:* based on the theory that all cognitive functions are believed to originate as products of social interaction, instructional practices emphasize the collaborative nature of learning (Vygotsky, et al., p. 57).
- *Guided instruction*: direct instruction which presents students with essential information as well as provides explanations of concepts and procedures or meaning. (Kirschner, et al., 2006).
- *Absolutism:* the view of mathematics based on the infallible, unambiguous truths and represents the unique realm of infallible knowledge (Ernest, 1998, p. 13).
- *Social constructivist philosophy* of *mathematics*: mathematical truths and the existence of mathematical objects must be established by constructive method. Human agreement is the ultimate arbiter of what counts as justified knowledge. (Ernest, 1998, p. 21, 48)
- *Meaning Theory*: a philosophical concept developed by Wittgenstein stating that meaning resides in social patterns of use which are woven into other aspects of social life (Ernest, 1998, p. 71).
- *Language- game*: a philosophical concept developed by Wittgenstein that compares language to a game, which are learned by participating in them (Ernest, 1998, p. 70).
- *Forms of Life:* A philosophical concept developed by Wittgenstein which explains that the speaking of a language is the connection of humans, a communal activity, the way of living in society (Ernest, 1998, p, 70).

- *Logic of Mathematical Discovery (LMD):* a unified theory of mathematical development, describing a method of teaching mathematics as a dialogue, in a "way that parallels history" (Ernest, 1998, p. 114).
- *Enactive mastery:* also, known as "performance accomplishments" (Bandura 1977, p.195).

Chapter 2

LITERATURE REVIEW

Introduction

In the previous chapter, the researcher presented some issues that relate to mathematics education in our nation over the past 30 years. In this chapter, the researcher will present further research into the sources, effects, and impact of teachers' negative mathematics attitudes, low efficacy beliefs, and mathematics anxiety. The proposed study emerges from a lack of research on the connection between self- efficacy beliefs and the use of social- constructivist instructional strategies. Therefore, research into the acquisition of quantitative literacy, mathematical self- efficacy, and the social constructivist theory of mathematics will be included. This chapter also includes a more detailed conceptual framework which will attempt to explain the significance of linking the above-mentioned theories to the problem of teachers' negative mathematics beliefs and attitudes, low mathematics self- efficacy beliefs, and mathematics anxiety. The researcher suggests that such understanding is needed to promote student understanding of mathematical concepts to thrive in the 21st century.

Review of Literature

The Relationship between Negative Mathematical Beliefs, Low Self- Efficacy and Anxiety

Positive attitudes toward mathematics are related to lower mathematics anxiety whereas negative attitudes have been observed to lower enjoyment and self confidence in mathematics (Hembree, 1990, p. 38). Negative mathematical attitudes of students develop through the attitudes and messages of parents, peers, teachers and society at large

(Boaler & Dweck, 2016; Vukovic et al., 2013). As stated in the previous chapter, many students, parents and teachers worldwide believe that math is a "dead subject, reserved for the smartest and cleverest people" (Boaler & Dweck, 2016, p. xii), only those with an innate ability can expect to well in it. "When students get the idea that they cannot do math, they often maintain a negative relationship with mathematics the rest of their lives" (Boaler & Dweck, 2016, p. x).

Research has shown that parents can exert indirect influence on students' mathematical beliefs (Vukovic et al., 2013). In 2014, Jameson found that a strong indicator of negative math attitudes and math anxiety is "self - perception of mathematics ability" (p. 533) which developed not from the home environment, but from the classroom. Some factors comprising a negative classroom environment are negative messages, hostile instructor behavior, gender bias, instructor inadequacy, ineffective teaching, difficulty of material and examination anxiety (Bekdemir, 2010; Boaler & Dweck, 2016; Geist, 2010; Jackson & Leffingwell, 1999).

Negative messages through the feedback students are given "can start students on a damaging and lasting mathematics pathway" (Boaler & Dweck, 2016, p. xi). Students in hostile environments characterized by negative feedback tend to self- organize in ways that reconcile the negative feedback with a more affirming view of themselves; in this case, a negative mathematical attitude. The students tend to devalue and often avoid the topic (Ravitch & Riggan, 2012, p. 124).

In the extreme, negative math attitudes can result in math anxiety, which is more than just a dislike of math. Mathematics anxiety has been described by researchers as a feeling of dread, tension, helplessness, insecurity and even panic relating to mathematics and mathematical tasks in academic settings and everyday life out of proportion with to the threat (Hayes, 2016; Hembree, 1990; Hughes, 2016; Maloney et al., 2013; Suinn & Winston, 2003; Weber, 2006). Math anxiety is a performance- based anxiety that has been studied for decades. It is not related to "general intelligence, and appears to be a cause versus a consequence of performance deficiencies" (Vukovic, et al., 2013, p. 449). This does not necessarily pertain to students with a diagnosis of dyscalculia, in which mathematics anxiety stems from a direct link to low achievement in math due to neuropsychological impairment (Rubinsten & Tannock, 2016). This would be valuable for further study.

Negative mathematical beliefs, attitudes and anxiety have been shown to have an impact on students' mathematical tasks due to the cognitive strategies involved. (Vukovic et al., 2013, p. 450). Negative feedback by repeated failure to complete complex tasks leads to increased anxiety (Harari, et al., 2013, Warwick 2008). Negative experiences within a classroom that go unchecked may lead to a "negative math perception spiral" (Jameson, 2014, p. 519), a self- fulfilling prophecy.

Some factors comprising a negative classroom environment are not merely negative messages, but also hostile instructor behavior, gender bias, instructor inadequacy, ineffective teaching, difficulty of material, and examination anxiety (Bekdemir, 2010; Boaler & Dweck, 2016; Geist, 2010; Jackson & Leffingwell, 1999).

Hostile instructor behavior can range from instructors making derogatory remarks, anger when students ask questions, and teachers punishing students for making mistakes (Bekdemir, 2010; Boaler & Dweck, 2016; Jackson & Leffingwell, 1999). Brain research has shown that mistakes are opportunities for brain growth as the student is challenged (Boaler & Dweck, 2016, pp. 15-19). Therefore, valuable, positive messages, when accompanying mistakes, can help students feel empowered. Class discussion about mistakes and solutions can lead to more positive attitudes about mathematics. Boaler and Dweck observed that "students feel really comfortable offering ideas and are not afraid of being wrong" (2016, p. 19).

Negative classroom experiences may begin as early as kindergarten, with mathematics anxiety- related behaviors being detected as early as second grade (Vukovic, 2013). Although research has been done to help treat the symptoms of mathematics anxiety (Hayes, 2016: Weber, 2005), in reviewing the above causes of negative mathematical beliefs, low mathematics self- efficacy and the resulting effects, mathematics avoidance and mathematics anxiety; the classroom environment, which includes instructional practices, teacher behavior, feedback and messages, must be addressed. Therefore, the topics of teacher preparation and practicing teacher instructional practices will be investigated.

In her meta- analysis into mathematics anxiety, Hayes (2016) mentioned that "for long- term relief, (of mathematics teaching anxiety), elementary education courses offer the most relief of mathematics anxiety, specifically the method classes that utilize manipulatives and include in service teaching experiences (Hayes, p. 97). The ways in which these factors improve teacher self- efficacy and lower their mathematics teaching anxiety will now be further discussed.

The Role of Mathematics Self Efficacy and Anxiety in Teacher Education

In 2016, Hayes reported that the mathematics anxiety in students may lead to mathematics avoidant behavior, even influencing choice of college majors which require less mathematics courses. Research has shown that many college- aged students with mathematics anxiety chose elementary education as their field of study (Hayes, 2016, Hembree, 1990, Phelps, 2010). These teachers, unfortunately, are now placed in a position to present a topic in which they do not feel comfortable with.

Pre- service teachers will eventually use their understanding of mathematics to teach others, and possibly pass mathematics anxiety and negative attitudes to their students (Boaler & Dweck, 2016; Beilock, Gunderson, Ramirez & Levine, Geist, 2010, 2015; Haciomeroglu, 2015; Harari, Vukovic, & Bailey, 2013; Jackson & Leffingwell, 1999). For pre-service teachers, a connection needs to be made between the college courses taken and the mathematics they will be teaching in their classrooms in order improve self-efficacy and motivation (Phelps, 2010). Hayes' study (2016) recommended that teacher preparation programs need to change their format to reflect an emphasis on more field work, which would include students observing teachers using manipulatives, reflective writing and mathematics discussion. Awareness of mathematics anxiety in students and its causes should also be a part of undergraduate elementary education programs (p. 102).

Research was conducted on the teaching practices of teachers whose pre- service mathematics education was based on constructivist perspective on learning (Sparrow & Frid, 2009). Most of the teachers in this study reported that their pre- service program supported their later endeavors, and "guided their practice" (Sparrow & Frid, 2009, p. 50). The teachers reported that they engaged students in using concrete instructional material as much as possible, used open- ended tasks, calculators and other technology and avoided textbook activities and worksheets. Textbooks were used as guidelines. The teachers also explained that they tried to develop a positive attitude in math through activities that were fun for students such as puzzles, games and projects. They developed a classroom specific curriculum which would cater to the development and achievement levels of their students. These teachers, although novices, were taking on curriculum leadership roles within their school by sharing expertise, strategies and resources with other teachers. In their interviews, several teachers reported that a key factor in developing their teaching practices was not only their pedagogical knowledge, but they knew how to implement it. They had not only a philosophy but also mathematics teaching portfolio. They were aware of a wide range of resources and strategies which could support mathematics learning.

The teaching practices and their relationship to improved teacher mathematics self- efficacy can help instructional leaders help practicing teachers promote student learning through professional development choices.

Teachers' Mathematical Self- Efficacy and Instructional Practices

Recent studies have found had conflicting findings about the role of mathematics anxiety and teacher instructional choices (Beilock, et al., 2010; Hughes, 2016; Jameson, 2014; Maloney et al.,2013), rather, the attitudes and perceived self- efficacy of teachers plays a key role in the classroom environment (Geist, 2015; Phelps, 2010). Teachers with low perceived self- efficacy will more likely teach in traditional ways, relying on textbooks to possibly alleviate anxiety (Hughes, 2016). It will also result in avoidance of topics and concepts within the mathematics curriculum for which they have the lowest self- efficacy or the use of teacher resources to avoid independent thought. (Geist, 2015; Hughes, 2016; Johnson & vandersandt, 2015). Research shows that teacher mathematic self- efficacy and mathematical learning goals promote student learning because they influence such factors as student effort and persistence. The reason for this is based on Bandura's extensive research and writing, which will be discussed further in this study. Briefly put, evidence shows that perceived efficacy beliefs contribute to effort and motivation (Bandura & Locke, 2003 p. 87). In Bandura's studies, his subjects shunned tasks that exceeded "perceived coping capabilities" (Bandura & Locke, 2003, p. 88).

Research cited in Phelps's (2010) article suggested that some factors that can influence students' motivational profiles are: verbal encouragement, which can either encourage or discourage the learner's mathematical self-efficacy; past performance, which can convince students of their abilities; classroom environment in which a constructivist approach can improve confidence; and "vicarious experiences"; that is, when a student sees other students performing an activity and then makes a judgement on their own ability (Phelps 2010). Learning goals are influenced by two main factors: social support and most strongly, classroom environment.

To elucidate, teachers with high mathematics self-efficacy beliefs use student mistakes to enhance their learning. They are more open to innovative ideas and methods "in order to meet the needs of students, compared to teachers who have lower self-efficacy beliefs" (Nurlu, 2015, p. 34). This type of feedback helps students improve their own mathematics self- efficacy.

In addition, highly efficacious teachers spent more time in academic activities and less time dealing with disciplinary issues (Bandura, 1997). In a study conducted by Ordonez- Feliciano in 2009, it was hypothesized that middle school teachers of mathematics with higher levels of self- efficacy used different instructional strategies than those with lower efficacy scores. In this study, they independent variable was self-efficacy, and the dependent variable was instructional strategies. Total efficacy scores ranged from 7.62 to 6.85, as measured by the Teacher Sense of Self- Efficacy Scale (p. 102). The hypothesis was partially supported. Data indicated a significant difference in teachers' use of problem based learning, manipulatives, and direct instruction. The mean differences respectively were .51, .46, .29 (p. 123).

Impact of Constructivist Instructional Models

Boaler & Dweck (2016) suggested that teachers need to present work, structure problems, guide students, and give feedback in ways which will promote positive mathematics self- efficacy. Based on their findings, Johnson and vanderSandt (2015) concluded that the current mathematics teacher preparation fails to prepare prospective teachers to meet the needs of diverse learners. They recommended both an increase in the use of concrete manipulatives and more focus on conceptual understanding in both the content and methodology courses. In addition, greater attention needs to be given to the needs of students with special needs (Johnson & vandersandt, 2015). Instructional methods for these students should be included to better prepare their future teachers and lower their anxiety, while also helping to promote "growth, innovation, creativity and the fulfillment of mathematics potential" in their students (Boaler & Dweck, 2016, p. xiii).

As previously stated, the culture of the classroom has an impact on students. A positive and supporting atmosphere can encourage learning. Research from multiple sources concludes that the students are greatly affected by the attitude that the teacher projects about mathematics (Bandura, 1991; Bergeson, 2000; Boaler & Dweck 2016; Cooper et al., 2012; Griggs et al., 2013; Gujarati, 2010; Krupa, 2011; Nurlu, 2015; Ulrich et al., 2014; Sparrow & Frid, 2009; Weber, 2006). Extremely negative experiences have been called “math trauma” (Boaler & Dweck, 2016, xii) leading to not only negative mathematical beliefs and low mathematics self- efficacy, but also mathematics anxiety.

As described earlier in this literature review, a positive and supporting classroom environment requires that teachers engage students in inquiry based learning activities and try to build a mathematical discourse within the classroom. The students should be building a personal relationship with mathematics built on their desire to explore mathematical concepts, and seeing the flexibility and creativity in mathematics (Boaler & Dweck, 2016, p. 59). Teachers must effectively scaffold the student’s prior knowledge with the desired learning outcome, using appropriate and engaging activities and expecting meaningful explanations of their work and reasoning (Boaler & Dweck, 2016; Powell & Kalina, 2009; Smith, 2004; Warwick, 2008).

Boaler and Dweck (2016) identified the five “C’s” of mathematical engagement (p. 57) which inspire excitement about mathematics: curiosity, connection making, challenge, creativity, and collaboration. Dr. Jo Boaler of Stanford University, California, challenges teachers (and provides sources) to create lessons that will inspire students and make teaching mathematics more responsive and creative, rather than traditional and assessment driven (Boaler & Dweck, 2016). She observed that “when students have learned norms of respect and listening, it is incredible to see the engagement when different ways to solve a problem are shared” (p. 59).

Pre- service course work can help prepare student teachers for the above-mentioned classroom experiences (Nurlu, 2015). Professional development for established teachers should extend over a significant period and enable teachers to develop a deep understanding of mathematics they teach and the ways that children learn mathematics. Effective professional development provides strategies for implementing pedagogies, and opportunities for reflection (Bergeson et al., 2000; Cooper, et al., 2012; Guskey, 2000; Krupa, 2011; Kuchey et al., 2009).

Past studies have attempted to analyze the attempted reform of mathematics instruction (Cooper, et al., 2012; Gabriele & Joram, 2007; Krupa, 2011; Powell & Kalina, 2009; Smith, 2004; Sparrow & Frid, 2009; Spillane & Zeuli, 1999; Ulrich, et al., 2014), by reflecting a complex understanding of instructional practices.

Spillane and Zeuli (1999) noted the "slow and erratic progress of reform in classroom practice" (p. 19), which is reflected in the issues discussed in the previous chapter. In their study, these researchers found that their sample of teachers reported "paying close attention to guidance about rethinking and revising their practices" (p.19).

The cognitive dissonance of attempting to revise practice and the failure to truly do so seems to be part of the reason why teachers dislike teaching mathematics. This lack of fit (Wyatt,2014) and its results will be the focus of the research presented in this study. In her study, Gujarati (2010) examined the link between teacher beliefs about mathematical thinking during classroom practice. One suggestion for further research in the study was to study teachers' mathematical beliefs and practices over time in order to "gain greater insight" (Gujarati, 2010, p. 275) as to what support teachers need not only in teacher preparation but once they are in their classrooms. The results of the current study can inform not only teacher preparation, but also professional development for in service teachers.

Theoretical Framework

Thus far, signs, symptoms and consequences of negative mathematical beliefs, low mathematics self- efficacy, and mathematics anxiety have been described; a physical description of the effects of mathematics anxiety on individuals does not how these thoughts, fears and beliefs have been produced. So, what is going on? In the previous section of this chapter several theories and concepts were delineated. First, the development of quantitative reasoning will be explored. Then, the social constructivist theory of mathematics, and its relation to self- efficacy will be discussed in order to explain how people bring about thoughts and actions, as well as generate "self- perceiving, self- reflecting, and self- reflecting activities" (Bandura. 1997, p. 5).

The Development of Quantitative Literacy-

Early quantitative literacy is more than "simple counting devoid of cognitive complexity" (Maclellan, 2012, p. 1). Number sense and related curriculum must be emphasized as part of early quantitative literacy. "Meaningful use of numerical information… must be a significant part of a teacher's practice" (Maclellan, p. 4). To do this, teachers must have strong pedagogic content knowledge of the elements that make up number sense. Professional development and pre- service teacher education should emphasize a deep understanding of the elements that make up number sense: number knowledge, counting skills and principles, nonverbal calculation, number combinations, story problems. Therefore, teachers can use reflection in action or professional noticing of students' mathematical thinking: attending to student behavior, interpreting student learning, and responding based on student understanding (Maclellan, 2012; Schon,1983).

Research has shown that the highest achievement comes when students learn through strategies rather than memorization; using visual and spatial representations for abstract facts allows both sides of the brain to communicate, enhancing learning (Boaler & Dweck, 2016, p. 39). Boaler and Dweck emphasize the importance of the development of number sense, the ability to work with numbers flexibly and conceptually (p. 35). They observed that when giving a problem to student before they were taught a method, students became curious. This curiosity "primed their brains to learn new methods (p. 66), and they learned concepts more deeply (p. 69).

In 2006, an article was published in the Journal Educational Psychologist which analyzed the failure of minimal guidance approaches to mathematics instruction, in which students must discover or construct information.

Kirschner (2006) discussed the problems of minimal guidance within inquiry based learning, which requires problem- solving of novices puts a burden on working memory, which is limited in duration and capacity. Therefore, new information acquired in this type of instruction can rarely be stored in long term memory. This means that learning will not take place. The article presents evidence from controlled experiments that when students learn in classrooms with pure discovery methods and little feedback, they often become confused and frustrated. This is especially true for novice learners. If students have little prior knowledge in an area, the load on working memory prevents understanding and learning. More effective use of inquiry based learning is to use inquiry along with instructional interaction and scaffolding procedures such as modeling and self-checking. Guided instruction, especially for novice to intermediate learners provided more immediate recall of facts but also, evidence shows, learning which can be applied to future problem- solving.

In contrast to the Kirschner study, Dr. Boaler found that students learned at significantly higher levels when taught conceptually, while collaborating, which enabled them to make connections to previously learned concepts. Induced curiosity seems to be the key to enjoying mathematics.

Social constructivism as a means of understanding the acquisition of quantitative literacy

Vygotsky's Social Development theory is one of the foundations of social constructivism. Social Development Theory states that consciousness and cognition are the result of social interaction. This perspective "regards individuals as inseparable from communities and environments" (Ravitch & Riggan, 2012, p. 36). In the last half of the twentieth century his work has been used by scholars and researchers to better understand cognition. This renewed interest in Vygotsky's work has been termed "neo- Vygotskian" and represents a "shift from thinking about learning as something that happens inside people's heads… to something that happens in the interactions among them" (Ravitch & Riggan, 2012, p. 84), which contrasts his work with that of Piaget.

Vygotsky, a teacher and Russian cultural psychologist was concerned with the relationship between humans and their environment, as well as the psychological consequences of activity. In addition, he explored the relationship between speech and learning, based on the interdependence of human thought and language. This relates to the development of quantitative literacy, in which each stage has three components: the number concept, the spoken word which represents the concept, and the written concept using symbols The spoken word "enables children to communicate their understanding of number concepts to others, and reflects their understanding of the number system and its rules" (Young- Loveridge, 1999, p. 2)

Vygotsky sought to reconstruct the changes in intellectual operations that normally unfold during the child's development. He also gave children tasks that exceeded their knowledge and abilities to discover rudimentary beginnings of new skills in the actual course of development (Vygotsky et al., 1978, p. 12). Vygotsky's theory differs from Piaget's model which "assumed that the child's mind contains all stages of future intellectual development, awaiting the proper moment to emerge" (Vygotsky et al., 1978, p.24).

In Piaget's model, development precedes learning. Piaget believed that the development of a child occurs through a continuous transformation of thought processes (Ojose, 2008, p. 26). Vygotsky does not refer to stages. In contrast, he argued that social learning, through interaction, precedes development. This argument is the basis for the need for students to converse in mathematics as they work through problems together. Even though the two theorists differ, one is impressed that there are periods of time that are necessary for optimal learning.

Piaget did not attribute a significant role to speech in the organization of the child's activities, nor stress communicative functions. Vygotsky places emphasis on the importance of language on learning. "The most significant moment in the course of intellectual development ...occurs when speech and practical activity converge" (Vygotsky et al., 1978, p. 24). Thought and language merge and produce "egocentric" thought or planful speech reflecting possible paths to solutions of a problem. In Vygotsky's research, it seemed necessary and natural for children to speak while they act.

"It plays a specific role in carrying out practical activity" (Vygotsky et al., p. 25). Using words to create a specific plan, while searching for and preparing strategies for solutions and planning future actions stimulates development. This planning function of language them to master their own behavior.

Vygotsky observed, through language, the child begins to master their environment. This mastery leads to a new relationship with their environment. Children solve practical tasks and problems with the help of their speech as well as their eyes and hands. "The more complex the action demanded by the situation, the greater the importance played by speech in the operation as a whole" and of such vital importance that if a child is not permitted to use it, the young children are cannot accomplish a given task (Vygotsky et al., 1978, p.25).

Another critical component in Vygotsky's theory is the role of communication with an adult. Asking for help from adult aids learning. Eventually, this problem- solving tool is turned inward. This internalization of speech promotes the development of the intellect. Learning has occurred. Egocentric speech emerges as the child begins to converse with himself in problems- solving. This leads to inner speech and logical thinking. (Ernest, 1998, p. 208). As discussed in the theory presented above, regarding the acquisition of numbers, the spoken word of numerical concepts can be the key to determining understanding by teachers.

Another important concept put forth by Vygotsky is the Zone of Proximal Development (ZPD), which is the difference between what children can do without assistance. This important to education because by providing the appropriate assistance to learners, we can enhance and foster skills and understanding that are emerging. The

Zone of Proximal Development is also referred to as scaffolding. Children, through interaction and imitation of peers, are capable of activities that are initially beyond their capabilities but within their range of competence. Learning occurs and awakens development of intellectual processes. Eventually, these processes become internalized. Fredrick Erikson (1996) observed that this interaction within classrooms is not necessarily a formal orderly dialogue between speaker and listener. It is sometimes messy…appearing as simultaneous participation which stimulates cognition (pp. 32-34; p. 51). Again, this theory supports the importance of students working in collaboration with teacher and peers to internalize understanding of mathematical concepts.

Vygotsky also researched the development of writing in children. Although he focused mainly on the development of writing and drawing in children, his writing on the topic applies to mathematics in that students learn to use symbols to represent quantification. As with writing letters to represent speech, learning to write numerals and other mathematical symbols is achieved through mastery of an "arbitrary combination of sign and meaning" (Vygotsky et, al., 1978, p. 117). In the context of numeracy, knowledge of numbers is different from the knowledge of quantities. This is the most abstract step for students, as they gradually grasp the connection between the spoken words and symbols (Young- Loveridge, 1999, p. 2)

Overall, Vygotsky stressed the importance that language and dialogue play in instruction and mediated cognitive growth emphasizing the need for guidance from adults and collaboration with peers (p. 131). In addition to internalizing learning through collaboration, learners can externalize and share with members of a group. Learning opportunities need to encourage "the learner's identity as skilled inquirer" (Ravitch &

Riggan, 2012, p. 36). In the field of mathematics instruction, this requires educators to shift from how they learned in school (Ravitch & Riggan, p. 35), which was based on the transmission of knowledge through a behaviorist perspective. This perspective views actions as the measurement of knowing. The teacher teaches, and the knowledge is received passively by students who perform actions which measure knowing (for example: memorization and speed tests), and motivation is extrinsic. The perspective of math as an absolute set of truths leaves little room for discussion, inquiry, creativity or collaboration. For instruction to change, the underlying assumptions about mathematics must also evolve (Ernest, 1997, 1998; Ravitch & Riggan, 2012).

Social Constructivist Theory of Mathematical Knowledge

Paul Ernest, of the University of Exeter, is a contributor to the social constructivist theory of mathematics. He argued against "absolutism" in the philosophy of mathematics, and called for reconceptualization of the field, which would encompass a shift from a behaviorist perspective, or even a Piagetian- constructivist view, to a social view, based on the work of Vygotsky. Mathematical philosophy is important; Ernest (1997) writes in that "any mathematical philosophy ...has many educational and pedagogical consequences when embodied in teachers' beliefs, curriculum development or examination system" (1998, p. 1).

In criticizing the traditional teacher- centered classroom model, Ernest (1998) described the underlying absolutist view of mathematics as "the source of the most infallible and certain of all knowledge" (p. 12); knowledge which is "timeless...superhuman and ahistorical" (1997, p. 2), presented logically with "necessary truths" (1998, p. 1) either generated from pure thought or empirical observation. A reflection of this view can be seen in instructional approaches which emphasize traditional, behaviorist, computational- based philosophies in which speed and accuracy supersede deep thought, exploration and inquiry.

Mathematical proofs form the basis for justifying mathematical knowledge. The proofs are based on propositions, based on previously stated axioms or rules (which are basic, self- evident truths). Therefore, these proofs are transmitting infallible truths. (Ernest, 1999, p. 8). In this view of mathematics, knowledge is a priori, and must be obtained from a source other than perceptual experience (p. 11). Is it possible to establish absolute truth in mathematics? Ernest sought to cast doubt on the infallibility of mathematical knowledge; “the certainty of mathematics cannot be established without making assumptions; this thereby fails to result in absolute certainty” (1998, p. 25). “Tautologies are true”, Ernest writes, “mathematics is not” (1998, p. 33). Ernest (1998) describes the weakening of the absolutist view of mathematics in the twentieth century. Even self- evident assumptions in one era can be scrutinized in another era. The outcome was the development of three major schools in the philosophy of mathematics, one of them being constructivism. The constructivist view of mathematics can trace its roots as far as Kant, who, in the late eighteenth century developed an elaborate system of philosophy in which mathematical knowledge (i.e., geometry) arose from the “unfolding of our intuition” (Ernest,1998, p. 20).

The constructivist view, therefore, establishes mathematical knowledge as having a personally meaningful nature. Mathematicians who promote the fallibility of mathematics call for a reconceptualization of the philosophy of mathematics, to promote it as a body of knowledge that is tentative and evolving. Absolutism adheres to a prescriptive accounting, programmatic, legislating how mathematics should be understood, rather than the nature of mathematics, descriptive of the history, objects and language of mathematics. An adequate philosophy of mathematics, according to Ernest, must include mathematical knowledge, theories, objects (signs and symbols), application, practice and learning. This last component must address how individuals learn as well as transmit knowledge (1998, p. 56). It must also emphasize individual creativity (Boaler & Dweck, 2016; Ernest, 1998). Mathematical knowledge cannot evolve without the human presence.

Ernest, (1998) credits the philosophy of Wittgenstein as a platform for the social constructivist theory of mathematical knowledge, one of whose concepts was meaning theory: that is, the meaning of a word or proposition is given by its use. He also wrote about the concept of the language games: rules of language are like rules in a game, and need to have an external goal (Ernest, 1998, p.70). These two philosophical concepts make up a larger theory, which Wittgenstein termed "forms of life". Language speaks one form of right and wrong, while actions may display another in all possibility either contradicting or supporting each other.

How does this apply to mathematics? Wittgenstein proposed a naturalistic and fallibilist social philosophy of mathematics. He was the not merely a philosopher, but the first mathematician to recognize the interdependence between language and mathematical knowledge (Ernest, 1998, p. 94). Mathematics is at once a branch of knowledge, but also a complex set of overlapping activities and language games. "There are no philosophical problems," writes Ernest, "only philosophical puzzles which can be sorted out by logical or linguistic analysis" (Ernest, 1998, p. 73). Mathematical certainty is grounded in accepted but reversible rules of a mathematical language games. This directly refutes absolutism in mathematics. Instead, mathematical knowledge is constructed by mathematicians; not discovered. Signs are given meaning by mathematicians, and do not preexist in a Platonic realm. New ones can be invented or added.

One aspect that Ernest finds lacking in the philosophy of Wittgenstein is the explanation of how mathematical knowledge grows and develops, for within his writings, there is only a description of mathematical language games justified by mathematical proofs, but does not account for the genesis of mathematical knowledge. (Ernest, 1998, p. 91).

For this, Ernest explores the philosophy of Polya, a mathematician who worked on problem- solving techniques. He stressed the "rational, publicly observable aspects of mathematical creation" (1998, p, 101) within a dialogue. His heuristic view of mathematics, among others, influenced Lakatos.

Lakatos criticized the teaching of mathematics as authoritarian. He sought to break down the division between informal mathematics and formal mathematical theories, broadening the scope of mathematics to be more descriptive of mathematical practice. (Ernest, 1998, p, 111). He is known for his Logic of Mathematical Discovery, or LMD, which describes in four stages, how a conjecture develops into a theorem. In LMD, the proof procedure is a dialogue. Lakatos showed that mathematical concepts, proofs and theories are contingent on a variety of circumstances, including the human powers of invention and criticism. Lakatos offers a method of teaching which he believed parallels the historical development of mathematical knowledge. By testing mathematical knowledge, a conjecture and proof are exposed to criticism, resulting in refinement and redefinition of mathematical concepts (Ernest, 1998, p. 118).

Ernest has taken these two philosophies, and used them to develop the social constructivist philosophy of mathematics, in which "objective knowledge" is that which is accepted as legitimately warranted by the mathematical community (1998, p. 147). Mathematical knowledge begins within the mathematical forms of life, as a conversation, either made face to face or written between participants. As claims are scrutinized, other participants help refine the claim. Eventually individuals use their knowledge to construct their knowledge and criticize and warrant the claim (Ernest, 1998, p. 149).

Knowledge, according to social constructivist theory, depends on language; it is rooted in conversation, the dialogical social process. This is a constructive act which may include counterexamples, counter arguments, and criticism of the proposal. As counter proposals are suggested and tried out, they may be modified or rejected. This is the basis for the fallibility of mathematical knowledge, "which never ceases to be open to scrutiny and revision" (Ernest, 1998, p. 154). Constructivists may argue that knowledge is an individual activity, guided by the individual's experience, Ernest sees this as an absolutist view which is known as "intuitionism". Mathematics is at its core, conversational. "The primary reality," Ernest wrote, "is conversation" (1998, p. 162).

Ernest's theory of mathematical knowledge is also built on Vygotsky's theory in which higher levels of thought develop as children internalize language because of interaction with adults, with the language games of Wittgenstein. "Thought is a form of internalized speech… a mental dialogue" (Ernest, 1998, p. 206). The social construction of knowledge in a teaching- learning context is dependent on "two- way participation in such conversations is… necessary to generate, test and validate mathematical performances (1998, p. 221). Conversation, and interpersonal negotiate generate and refine mathematical knowledge. Spillane and Zeuli, (1999) wrote that "through conversation about mathematical ideas, students not only learn from one another, but also bring to the surface insights and understandings that are not possible otherwise (p.5). Erikson (1996) noted that simultaneous participation within classrooms is cognitively stimulating as students clarify and model reasoning (p. 51).

The nature of play and language as applied to Ernest's theory is that play enables children to attach alternate meaning to concrete objects. This is the genesis of symbolic thought. Transformation of signs and symbols is essential to mathematics. These artificially contrived signs and symbols are "thinking devices" (p. 221) that must be socially acquired and mastered. "Mathematics is learned through participating in language games, embedded in forms of life" (Ernest, 1998, p. 220). Sustained two- way participation in conversation is necessary to generate, test, correct and validate mathematical performance" and ensure that "the learner has appropriated the collective mathematical knowledge and competencies...not some distorted version" (p. 221). The attainment of knowledge is dependent on the opportunities for "individuals to participate in the practices of communities (e.g., the mathematics community)" (Ravitch & Riggan, 2012, p. 36).

In her 1999 framework established for the acquisition of numeracy, Young Loveridge observed that students who were given the opportunity to participate in activities using concrete materials which were based on real life activities made more progress in reaching multi- unit understanding of numbers. She also suggested that children be encouraged to invent their own strategies for solving problems. In fact, she stated that “invented strategies are thought to provide a useful context for advancing a useful context for understanding” (p. 6). Ernest (1998) stated that imposed tasks requiring students to carry out symbolic transformation do not allow the social context of learning, the negotiation between learner and teacher. Writing out steps and labeling an answer does not match the learner’s process in deriving the answer. Problem- solving and investigational (discovery) learning requires that students describe judgements, conjectures and thought processes involved in a mathematical subject (p. 225). As Boaler and Dweck (2016) observed, not only does collaboration and discussion with the mathematics classroom enliven the subject and engage students, but helps the students develop mathematical reasoning and critique others’ reasoning (p. 29).

The result of the use of discourse, collaboration and scaffolding within mathematics instruction will be a social more positive classroom, in contrast to what Ernest describes as more traditional, or transmission- based: “rigid, fixed, logical, absolute, inhuman, cold objective, pure abstract, remote and ultra- rational” (1997, p. 2).

Mathematics self- efficacy

“People will approach, explore, and try to deal with situations within their self- perceived capabilities, but they will try to avoid transactions with stressful aspects of their environment they perceive as exceeding their ability” (Bandura, 1977, p. 203).

Self-efficacy is a component of social cognitive theory, being a prime determinant of self- regulatory activities affecting thought, affect, motivation and action (Bandura, 1991, p. 257; Martocchio, 1994, p. 820). People’s beliefs in their efficacy influence the choices they make, how much effort they expend and long they persevere in each activity (Bandura & Locke, 2003).

Bandura (1997) explained that “knowledge structures are formed by observational learning, exploratory activities, verbal instruction, and innovative cognitive synthesis of acquired knowledge" (p. 34). Self- efficacy is formed by interaction of personal factors (cognitive, affective and biological events) and the external environment (Dellinger, et al., 2008). This causal model between self and society is a "triadic reciprocal relationship… personal factors and the environment influence behaviors, while the environment is impacted by behaviors and personal factors, and personal factors are impacted by the behaviors and the environment" (Dellinger, et al., 2008, p. 752).

In his book, Bandura (1997) explained how self- efficacy is formed and changed. People use these knowledge structures to execute actions: skill eventually become easily executed. People with the same skills may perform poorly or adeptly, because their efficacy beliefs affect how well they use their skills. Self- efficacy is concerned with what an individual believes they can do with the skills they have. Efficacy beliefs guide behavior and are reappraised when conditions are altered. The ability to envision the likely outcome of a course of action leads to planning, foresight and adaptation, influencing motivation. Perceived self-efficacy affects the planning of action and motivation by forming and shaping aspirations and directing the use of skills (Bandura, 1997, p. 35). Poor efficacy beliefs can undermine performance; skill can be overruled by self-doubt whereas a more resilient sense of self efficacy enables a productive use of skills. Those who persist in perceived threatening situations which are relatively safe will gain "corrective experience" (Bandura, 1977, p. 194). For instance, being helped and encouraged to help a person feel more capable. Ceasing prematurely will "reinforce debilitating behavior" (p. 194). Those with strong efficacy beliefs tend to set higher goals as they attain a standard of achievement they have been pursuing (Bandura, 1991,

p. 260).

Mathematics self- efficacy, which can be described as a personal cognition, “a measure of the student’s belief that they can, in each situation, successfully complete a particular task” (Warwick, 2008, p. 32) that being mastery of concepts for applicability, and is formed at the elementary stage of education.

The individual makes self- efficacy judgements based on four main sources of evidence. The first is performance assessment. This is important because they tend to extinguish fear arousal or preclude fear from elevating to toxic levels, thus authenticating through enactive mastery sources of information about one’s capabilities for coping. Repeated failure early during events lower expectations. “Once strong efficacy is established occasional failures are overcome by determined effort and strengthen motivation and can be generalized to other situations, even those that are substantially different, but most predictably to those most similar” (Bandura, 1977. p. 195). Bandura described participant modeling as participants gaining opportunities to practice appropriate actions, by watching a preliminary performance followed by graduated tasks and joint performance It is important to provide opportunities for “self- directed accomplishments after desired behavior has been established (Bandura, 1977, p. 201) in order to authenticated personal efficacy and insulate it from disconfirming evidence (p. 202). Generalized, lasting changes to behavior can be achieved through powerful induction to develop capabilities, removing external aids, and the use of self- directed mastery, i.e., independent performance. This relates to Vygotsky’s observation that the Zone of Proximal Development allows students to internalize activities once just beyond their capability through interaction and imitation. As previously discussed, this also supports the use of more scaffolding, discourse and collaborations in mathematics instruction.

The second source of efficacy beliefs is “vicarious experience” which means comparison with peers. This source is weaker than direct evidence from personal accomplishment, therefore more vulnerable to change. “Seeing others perform threatening activity without adverse consequences can generate expectations in observers that too will improve if they intensify and persist in behavior” (Bandura, 1977, p. 197). It has been shown that people with low efficacy benefit from observing others overcome difficulties more than observing experts easily perform a task. This helps observers develop a sense that they can succeed also.

The third source of efficacy is “verbal persuasion” which means comments made by people in authority, such as teachers and parents. It is also feedback on work completed. This is a widely-used form of influencing behavior of others.

The fourth source of evidence which contributes to self- efficacy is emotional arousal, or “physiological and affective states” which means the feelings of anxiety and worry or happiness, confidence in the positive orientation (Bandura, 1977, p. 191; Warwick, p. 32). Individuals are more likely to expect success when they are not beset by aversive arousal than if they are tense and agitated. By conjuring up fear- provoking thoughts about ineptitude, individuals can rouse themselves to elevated levels of anxiety that far exceed the fear experienced during the actual threatening situation (Bandura, 1977). Avoidance of these stressful activities interferes with the development of coping skills. The resulting lack of competency provides a “realistic base for fear” (p. 199). The informative value of emotional arousal depends of the meaning imposed on it.

Individuals who believe that their arousal is due to personal inadequacies are more likely to lower their efficacy expectations resulting in “reciprocally escalating arousal” (p. 202).

In his 1977 article Bandura stated, “people who regard outcomes as personally determined, but who lack skills would experience low self-efficacy and view activity with a sense of futility” (p. 204). There are two ways of considering ability, as a fixed entity, or an acquirable skill. Those who view ability as a fixed entity focus more on “evaluative concerns about personal competence… they tend to become more self-diagnostic than task diagnostic” (Martocchio, 1994, p. 820).

To illustrate this, Bandura (1977) used an example of a child learning mathematics becoming “demoralized” because they have failed to grasp concepts and believes that their grades will reflect their lack of skill (p. 204). “People can give up trying because they lack a sense of self- efficacy in achieving the required behavior, or they may be assured of their capabilities but give up because they expect their behavior to have no effect on an unresponsive environment or to be constantly punished” (Bandura, 1977, p. 205), in this case with poor grades. Students may also be making social comparisons with other classmates.

It had long been believed that self- efficacy beliefs were most strongly altered by enactive modes due to the performance. Mastery of challenging tasks conveys salient evidence of enhanced competence. Detecting progress even when experiencing setback will raise efficacy more than those who see performance leveling off (Bandura, 1977). Efficacy expectations induced by verbal persuasion were thought to be weaker than from one’s own accomplishments because they do not provide an authentic experiential base; and efficacy expectations can be easily extinguished by disconfirming experiences

(Bandura, 1977, p. 198).

In contrast, Bandura and Locke (2003) found that past performance is only a measure of what a person has done, not always a predictor of what a person can do in the future. Persuasion and comparison are more influential on perceived efficacy, even if these are based on erroneous or illusory feedback.

In research presented by Boaler and Dweck, students who saw ability as an acquirable skill (or having a growth mindset), demonstrated more brain activity following mistakes than for students who believed that ability was a fixed entity (those with a fixed mindset). (p. 12). Therefore, there is physical evidence that self- efficacy impacts learning in students.

In addition to the above-mentioned example of physical evidence, Bandura presented evidence in which neutral stimuli, when associated with painful experiences, create the anticipation of aversive consequences (Bandura, 1977, p. 209). Therefore, it is not the mathematical tasks that have become aversive, but the association with painful experiences such as poor grades, embarrassment, confusion. Stimuli have a predictive significance and signal consequences unless predictive measures are taken. Extinguishing anxiety arousal using methods such as visualization, expressive writing, or verbal persuasion is rarely a sufficient condition for eliminating defensive behavior because it is only one source of efficacy information (Bandura, 1977, p. 212). People fear and avoid situations which are threatening to them. Therefore, in the case of mathematical tasks, individuals with math anxiety will avoid situations which they believe are beyond their coping skills and are therefore intimidating. Avoidant behavior is a result of perceived threats, which overtime, may further breed failure. Bandura

(1991) explains that people naturally expend less effort on devalued activities which affect their welfare and self- esteem (p. 255). As evidenced in Bandura's extensive studies, perceived efficacy beliefs directly affected motivation (Bandura & Locke, 2003, p. 88).

In contrast, individuals who see ability as acquirable skill approach learning with the belief that "capability can continually be increased by gaining knowledge and building their competencies through practice (Martocchio, 1994, p. 820). Learning is not a threat, but an opportunity to develop new strategies and skills to complete a task.

The link between mathematics self- efficacy and student engagement and ways that self- efficacy can be enhanced was studied by Warwick, in 2006. This qualitative study found that mathematics self- efficacy can be considered alongside mathematics anxiety in relation to mathematics performance.

There is a direct link between student engagement and mathematics self- efficacy. There are three types of engagement, and they all directly influence self- efficacy beliefs (Warwick, 2008). The first is behavioral engagement which can be explained as student's efforts, their interaction with teacher and peers, their willingness to seek help, attendance in class. "High levels of self-efficacy are likely to encourage perseverance in the face of difficulty. Low efficacy beliefs result in less likelihood of students asking for help. In their 2003 study, Bandura and Locke found beliefs of personal efficacy contributed to willingness to perform a threatening task (p. 88).

Cognitive engagement means "minds- on". A student appearing to work on a mathematics problem is not necessarily indicative that the student fully engaging mental faculties in trying to complete it" (Warwick, 2008, p. 32). Cognitive engagement is a result of the way a class session is structured, and the way the teacher interacts with the students. If a student feels that they can complete a task, then they will more likely engage and persevere with appropriate cognitive strategies (Warwick, 2008).

The third type of engagement is “motivational engagement”. That is, the student has personal interest, feels that it is useful, and generally important to learn (Warwick, 2008).

The important question posed is “How can we use classroom practice and curriculum design to enhance self- efficacy and student engagement so as to generate positive reinforcement?” (Warwick, 2008, p. 33). Specific feedback, reflection, and emphasis on real world connections will help reduce anxiety and improve self- efficacy and engagement. This can “significantly improve student performance” (Warwick,2008, p. 36).

Teacher Efficacy and Mathematics Education

Bandura (1997) states that teachers’ perceived efficacy “rests on much more that the ability to transmit subject matter” (p. 243). It also includes their ability to maintain an orderly classroom, conducive to learning, encouraging parental environment, and counteracting negative social influences. Gabriele and Joram (2007) stated that teacher efficacy is the “effect of efficacy beliefs on the motivation to expend effort, on the willingness to set challenging goals, and on the persistence through difficulty (p. 60). Negative effects of low efficacy may cause “burnout”, which Bandura (1997) describes as a syndrome of reactions to “occupational stressors”, resulting in physical and emotional exhaustion, depersonalization and demoralization (p. 242). Bandura and Locke (2003) have recommended further study into under- confidence and the “selfhandicapping costs of self- doubts about one's capabilities" (p. 97).

Teacher efficacy has a strong influence on young children because their own beliefs are still "unstable... and make little use of social comparison in evaluating their capabilities (Bandura. 1997, p. 242). Students with low efficacy beliefs are vulnerable to negative effects of teachers with low efficacy; they suffer declines in their expectations of academic performance. Students with low efficacy beliefs tend to expect more of themselves when placed with teachers with high efficacy beliefs (Bandura. 1997, p. 242). Personal standards are developed from information conveyed by those around us. Teachers not only teach and prescribe standards for their students, "they exemplify them in their reactions to their own behavior" (Bandura, 1991, p. 254).

Teacher self- efficacy beliefs are task and situation specific, learned and active varying in strength, level and generality (Dellinger et al., 2007). Mathematics education is a specific facet of teaching. Therefore, teachers may have low efficacy beliefs for only mathematics or for only one aspect of mathematics. Efficacy beliefs may vary in level according to the perceived difficulty of the task due changes in student characteristics; for instance, a teacher may have distinct levels of efficacy beliefs about teaching an aspect of math to honor students compared to students in a regular classroom (p. 754 and p. 761). Also, efficacy beliefs about teaching mathematics may carry over or generalize to similar activities.

Teacher efficacy is a powerful influence on teacher learning. Persistence and effort are intensely affected by efficacy beliefs (Bandura, 1977, p. 212). "Beliefs of personal efficacy constitute the key to human agency" (Bandura, 1997, p. 3); that is people will not attempt to act if they do not believe that they do not can produce results.

It is the key factor in initiating and sustaining changes.

Low self- efficacy will also include emotional withdrawal, and disengagement from instructional activities (Bandura. 1997, p. 242). An important source of self-efficacy is reflection of past performance. This is related to Bandura's above- mentioned theory in which the formation of self- efficacy beliefs is based on reflection and interpretation of past performance, which is known as enactive mastery experiences (Bandura, 1977; Gabriele & Joram, 2007). Gabriele and Joram (2007) explain that when there are changes in teacher's ability to see evidence of success, teacher efficacy is improved (p. 62). The use of social constructivist methods which focus on children's thinking and strategies may not provide the traditional means of evaluating student success. Gabriele & Joram (2007) examined the shift in criteria used by teachers to evaluate their success in teaching. The researchers asked teachers to reflect on lessons. The results of this study showed that teachers who were "newcomers" or less experienced teachers focused on their performance, while more experienced, or "veteran" teachers focused on student thinking. Student- focused reflection led to higher teacher self-efficacy for using "reform-based" constructivist mathematics instructional methods (Gabriele & Joram, 2007, p. 71).

Ernest (1998) described the teacher's role as mathematician is to transmit knowledge through social interactions. If a teacher adheres to the absolutist philosophy of mathematics, this would be a one-sided transmission; whereas a teacher whose philosophy adheres to a fallibilist philosophy of math will most likely focus on student thinking.

Self Determination and Reflective Practice

Bandura states that “the choice of action is not completely and involuntarily determined by environmental events” (1997, p. 7). Making choices is enhanced by reflective thought. Reflection, which Bandura describes as the “capacity to exercise self-influence by personal challenge and evaluative reaction to one’s attainments” (1991, p. 260), provides a basis for goal setting, thereby enhancing motivation. Self- motivation comes from self- challenge toward a goal with evidence of progress towards a personal goal (Bandura, 1991, p. 263). Self- evaluation provides evidence of progress which enhances performance. “Satisfaction in personal accomplishment becomes the reward” (Bandura, 1991, p. 265).

In the writing of Sch n (1983), reflection most optimally takes place in practice. Conversation enables a teacher to analyze the cognitive progression of their students. They can therefore take immediate action to help their students. This is called “reflection in action” or professional noticing. Teachers notice and try to make sense of student’s mistakes. “In each instance, the practitioner allows himself to experience surprise, puzzlement, or confusion” (p. 68) and carries out experiments which will generate new understanding and change the situation. This responsive model is enabled by the instructional model which incorporates inquiry, collaboration and conversation in which students simultaneously respond to mathematics interactively with their peers and teachers (Boaler & Dweck, 2016; Erickson, 1996; Ernest, 1997, 1998; Gabriel & Joram, 2007; Sch n, 1983).

Sch n (1983) described students reflecting on problems presented them, learning to take initiative in solving them for themselves, and to settle disagreements within a group by experiment and most importantly, learned to "model the unfamiliar on the familiar and to reframe their questions around the changes which resulted unexpectedly from their actions" (p. 201). Students were given control over their learning using concrete, dramatic contexts which would capture the students' attention, encouraging discussion and collaboration. Reframing the pedological view of knowledge from one based on transmission to one of communication and reflection can thereby lead students through a process of inquiry which clarifies understanding and demonstrates a mode of thinking about problem solving (p. 316). In this model, a teacher tries to find out, by really listening, what the students are thinking, what the sources of confusion could be, thereby inventing new activities and questions for students… and new ways to help them learn (p. 332). Gujarati described this process as "building new understandings to inform one's actions of the situation unfolding" (p. 24). To do this, teachers need "the freedom to reflect and invent" (p. 333) as well as communicate with peers to explore and test insights.

This type of learning experience is like the process of learning described by Ernest. The genesis of mathematical knowledge is through questioning, probing and exploring, reframing questions, testing and validating mathematical performance. A teacher's expertise can be considered "a way of looking at something which was once constructed and may be reconstructed" (Sch n, 1983, p. 296) as Ernest describes the LMD- evidence is discussed and refuted or accepted: each participant trying to understand what the other is experiencing.

Conceptual Framework

Teachers play a key role in forming the mathematical attitudes of students. Teacher beliefs toward mathematics and their beliefs in their ability to organize and execute their teaching affects student attitudes and achievement (Nurlu, 2015). It has previously been established that the perceived self- efficacy of teachers will determine the choices they make in lesson- planning and instructional practices (Bandura & Locke, 2003; Gujarati, 2010; Hughes, (2016). The attitudes and beliefs of the teacher affect the classroom environment. The factors which make up a social constructivist classroom: discussion, collaboration, and inquiry enable the teacher to assess their students' understanding of concepts (Gujarati, 2010). This feedback allows teachers to reflect- in-action (Sch n, 1983) and improves teacher perceived self- efficacy through what Bandura called "verbal persuasion" (1997).

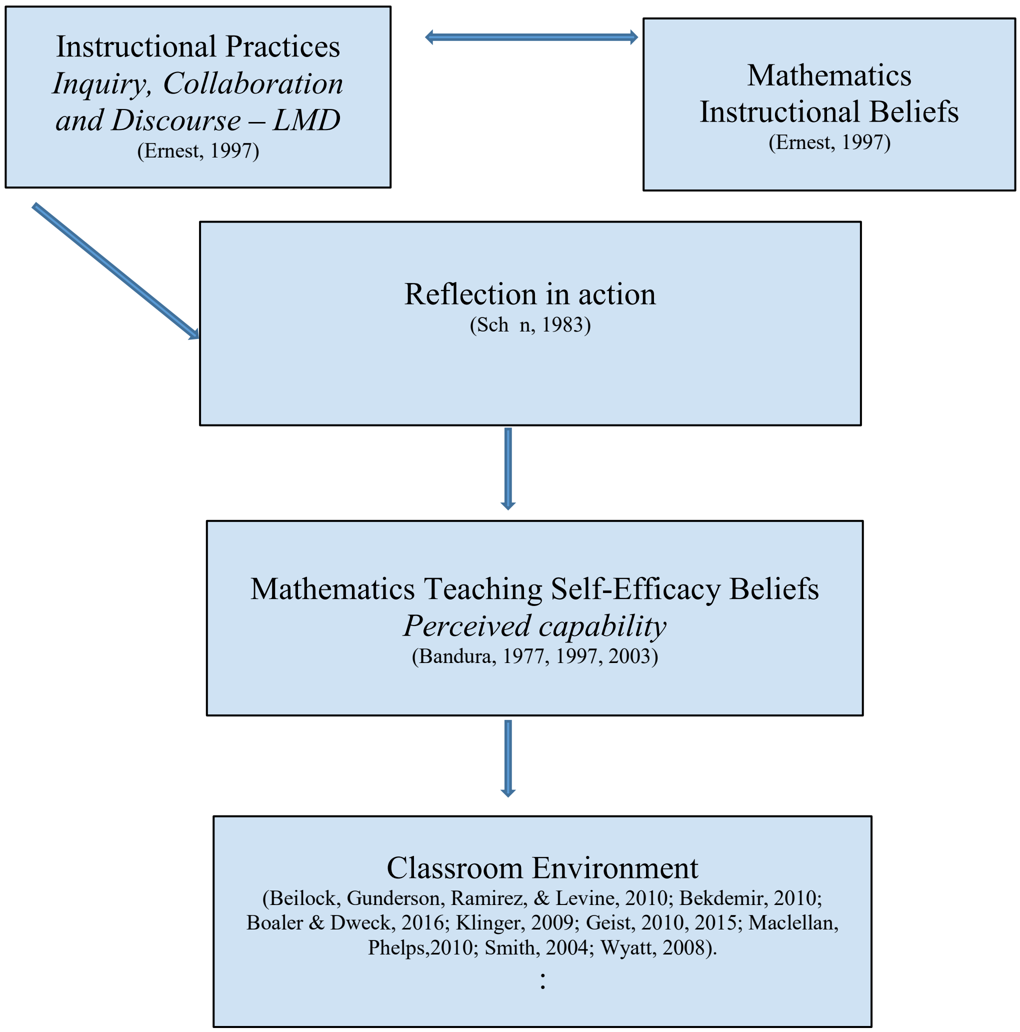


Figure 1: Conceptual Framework. This figure shows the relationship between mathematics instructional beliefs, mathematics instructional practices, mathematics teaching self-efficacy and classroom environment.

Summary

Spillane and Zeuli (1999) wrote that "teaching is a multidimensional practice" (p. 19). Reform- based teaching requires not only more student-centered activities, rather, teachers must recognize and support "new conceptions of knowledge and knowing" (Spillane & Zeuli, 1999, p. 19). Ernest (1997) wrote about the pedagogical implications of moving from an absolutist view of mathematical knowledge to a fallibilist one. Similarly, Sch n (1983) wrote about the demystification of professional knowledge by opening it up to inquiry. In the traditional model of education, teachers are "experts who impart privileged knowledge to students… (who) are fed portions of knowledge in measured doses" (Sch n, 1983, p. 329). As this knowledge becomes open to inquiry, the relationship between teachers and students "takes the form of a literally reflective conversation… in which the teacher's expertise is embedded in a context of meanings" (Sch n, 1983, p. 295). In this model students are assumed to have the capacity to mean, know and plan. By becoming curious about student reasoning and behavior, teachers overcome the feelings of shame and vulnerability associated with students' deficient performance and thereby help the students think their way through problems overcoming their own fear of failure (Sch n, 1983, pp. 321- 322).

The following study will investigate the effect instructional practices on the self- efficacy beliefs of teachers.

CHAPTER 3

METHODOLOGY

Introduction

The following chapter describes the research methods for this quantitative study. The purpose of this study is to improve the educational experience of our youth. The effort to improve teaching is not currently unified in a matter of course. Research seems to be at the periphery of deeper understanding. How can pedagogy be used to enhance learning? What is effective teaching? What does it look like? The programs offered by current trends offer mild solutions. Teachers hear catch-phrases such as “engagement”, ‘manipulatives”, “hands- on”, and “student- centered”. Do they really understand the learning process and how the above- mentioned strategies should be put in place?

Statement of Purpose

The purpose of this study is to assess the relationship between instructional practices and teacher mathematical self-efficacy beliefs as measured by the Mathematical Self- Efficacy and Teacher Instructional Practices Survey. Previous studies have found that teacher mathematical self- efficacy was “related to important skills teachers need to motivate their students” (Ordonez- Feliciano, 2009, p. 134). Further research is needed to help administrators enhance teaching practices of teachers as they gain experience (Enochs, Smith, & Huinker, 2000; Ordonez- Feliciano, 2009). The results of this study will help guide administrators plan such training options for teachers.

Rationale for Research Approach

A quantitative design was chosen for this study, to provide data for quantitative educational leaders, whose role, Bowers (2016) explained is to “focus on translating data analysis information for evidence based improvement" (p.88). Correlational analysis will be used to measure the degree of association between self- efficacy and instruction, as expressed by a number which will indicate whether these two variables are related or whether one can predict the other (Creswell, p. 2015, p. 21).

Previous studies have explored teacher- self- efficacy in mathematics. In 2009, Ordonez- Feliciano compared middle school teachers' self- efficacy scores using the Ohio Tate Teacher Efficacy Scale (TSES), created by Tschannen- Moran and Hoy in 2001, and instructional practices using the Teacher Instructional Survey created by Hass by Haas in 2002. The purpose of the study was to determine whether self- efficacy of mathematics teachers was related to their choice of instructional strategies (p. 119). The dependent variable was the instructional practices of teachers. Whereas Ordonez – Feliciano found that instructional strategies used by the low self- efficacy teachers were significant in the areas such as problem- based learning, manipulatives, multiple representations and direct instruction (p. 126), post- hoc tests to compare factor significance of teachers' instructional strategies and mathematics teaching self- efficacy groups were not performed (p. 114).

The current study will further test the relationship between instructional strategies and self- efficacy beliefs to more fully understand the relationship between the two groups, and find specifically what most affect mathematics teaching self- efficacy beliefs of teachers. As noted by Ordonez- Feliciano (2009), benefits of self- efficacy include "reflection, motivation to learn, greater response to diversity, productive collaboration (p. 124). Data from the current stud can be used to guide professional development as well as pre- service elementary mathematics curriculum (Enochs, Smith & Huinker, 2000; Ordonez- Feliciano, 2009).

Research Questions

1. What are self- efficacy beliefs of teachers of elementary mathematics?
2. What instructional strategies characterize those of elementary mathematics teachers?
3. What are teachers' beliefs about student learning and mathematics instruction?
4. What is the relationship between reflection in action, classroom discourse, and teacher self- efficacy in mathematics instruction?

Setting and Context

This study takes place in the Diocese of Rockville Centre, located on Long Island, New York. Established in 1957, the Diocese encompasses 1198 square miles in Nassau and Suffolk County. There are approximately 32,000 students engaged in Catholic education, 16,000 of these students attend 47 elementary schools (Bishop's Advisory Committee (BAC) 2011).

In the Diocese of Rockville Centre Strategic Plan for Catholic Elementary Schools (BAC, 2011), the Executive Summary states that the goal of the Bishop's Advisory Committee and Diocesan Education Department is to "develop and foster the implementation of a comprehensive strategic plan to support the long-term sustainability, growth, and excellence of Catholic elementary schools on Long Island". Six main goals are highlighted in the report: strong Catholic identity, effective organization and governance, collaborative leadership, responsible stewardship, vibrant and effective communications, and academic excellence. The results of this study can help the Diocese maintain the highest standards of performance and improve academic programs (BAC, 2011).

Sample and Data Source

This research will use convenience sampling. According to Creswell (2015), convenience sampling includes participants that are “willing and able to be studied” (p. 144). In this case, the sample will be elementary mathematics teachers in the Diocese of Rockville Centre who respond to the survey. The Google form used for the survey has been designed to be anonymous; it cannot collect names or email addresses of respondents.

To ensure confidentiality, the researcher e-mailed the invitation to participate in the study to the Superintendent of Education for the Diocese of Rockville Centre. After receiving approval from the Diocese, the invitation and survey was disseminated to teachers through School Messenger, a communication system which is designed to reach all the teachers in the Diocese.

Data Collection Method

This research study will use a quantitative design to support its data. The quantitative component is based on descriptive statistics and the relationship between variables (Creswell, 2015). This quantitative study seeks to establish relationships and generalizations from collected data (Muijs, 2011).

The survey used for this study has been adapted from the Instructional Interactions Survey (IIS) used by the Distributed Leadership Study (DLS), conducted by Northwestern University School of Education and Social Policy for NebraskaMATH, and is sponsored by the National Science Foundation. The survey was originally used by Hopkins and Spillane in 2013 and 2015, to measure the relationship between instructional guidance infrastructure and teachers’ beliefs about how students learn mathematics. The

IIS focuses on sources of leadership for mathematics and language arts instruction in elementary school. It asks questions about instructional strategies and teachers' beliefs about teaching and learning in general, and mathematics in particular. It also asks information about professional interactions between teachers and educational leaders. Dr. Spillane of Northwestern University granted the researcher permission to use the survey (see Appendix A).

For this study, the researcher will only use questions regarding instructional practices, and teacher beliefs about teaching and learning mathematics. After developing a conceptual framework, the researcher coded the survey question on the IIS in NVivo11, focusing on the components mathematics self- efficacy, reflection in action, collaboration, inquiry, and classroom discourse. After the survey was compiled, it was coded again in NVivo11 to ensure that the items on the survey reflect the abovementioned components (Appendix F).

To instructional practices, 11 items on the survey use a 4-point Likert scale, with 4 labels denoted as 1= Never or almost never, 2= Some lessons, 3=Most lessons, 4= Every lesson. These questions measure student tasks, inquiry, collaboration and discourse within instructional practices.

Eighteen items additional items were included to measure teachers' beliefs about best instructional practices and student learning: specifically, student tasks, collaboration, inquiry, and classroom discourse. Fourteen items were included for measuring self-efficacy beliefs regarding the teaching of mathematics. Five items were also included to measure reflection in action. The survey uses a 5-point Likert scale with labels denoted as 1= Strongly agree, 2= Disagree, 3= Neutral, 4= Agree, 5= Strongly agree (Appendix

E). Some consideration was given to whether the scale should include a neutral position. Muijs (2011) suggested that using a neutral position may result in central tendency problem, in which respondents choose this option for sensitive or controversial questions. However, by eliminating the neutral position, the survey may therefore misrepresent the views of respondents who are truly neutral about some questions (p. 42). Therefore, for the neutral position was included for items measuring self- efficacy belief regarding the teaching mathematics. After the respondents fill out the survey the researcher will reverse score the negative items, so that the individual item scores lie on the same scale with regard to direction (Appendix E)

The researcher will conduct a correlational analysis to determine if the relationship between variables is statistically significant. Correlation coefficients will be computed to describe the direction and strength of relationships.

Issues of Trustworthiness

The survey used was compiled from instruments validated by prior research (Ravitch & Riggan, 2012, p. 117). As stated by Tavakol and Dennick, (2011, p. 53), “internal consistency should be determined before a test can be employed for research or examination purposes to ensure validity”. Therefore, the researcher examined the internal consistencies as reported in two articles in which the results of the IIS were discussed.

The original IIS was used in two school districts in Illinois, Auburn Park and Twin Rivers. In Auburn Park, 331 school staff members responded in 2010, 393 in 2011, and 384 in 2013, which was a response rate of 81, 95 and 94% respectively. In Twin Rivers, 243 staff members responded in 2010, 276 in 2011, and 316 in 2013, with a response rate of 68, 71 and 83%. Most respondents were full- time teachers assigned to single grade levels (Hopkins & Spillane, 2015). The survey contained 18 items related to math teaching. Based on factor analysis of these items, the researchers found two factors, one related to teachers' beliefs about student learning (Cronbach's alpha = .86), and another related to how best to teach math (Cronbach's alpha = .82) (Hopkins & Spillane, 2015, p. 429). Additionally, these 18 items on the survey were used to measure teacher attitudes and beliefs about teaching mathematics (Cronbach's alpha = .83) Six items were also used to measure practices (Cronbach's alpha = .73) (Hopkins & Spillane, 2013, p. 205).

The Cronbach's alpha reported above is the property of the sample used in the Auburn Park and Twin Rivers study (Hopkins & Spillane, 2013, 2015). Therefore, the researcher will measure the alpha once the present survey is administered.

Limitations of the Study

Observations and interviews can further explore if practices align with self- report and impact on self- efficacy, as well as capture nuances of interactions and feeling which a survey cannot. The respondents reflect a certain type of teacher, which may provide a certain bias for or against the goal of this study. Since the researcher is not studying the type of teacher who would take the time to answer the survey, the implications of this bias cannot be measured. It is a limitation of this study. A conscientious teacher who is punctual with their work and reports may also be more likely to respond to a survey. Conversely, a conscientious teacher may not have the time to take a survey which is not mandatory. Since we cannot know this, the researcher cannot assess the bias of the data collected.

The survey did not collect data regarding the years of experience of the respondents, and this could be a factor which influence the factors being explored in this study. Furthermore, although the survey collected data regarding the grades taught by the respondents and the gender of the respondents, this was not within the scope of the current study.

Finally, the small sample size, as well as the sample chosen is a limitation of this study. The survey was given to teachers within the Diocese of Rockville Centre, located in Nassau and Suffolk County, New York. The results, therefore, may not be transferable to other districts. Until a study is conducted outside the Diocese of Rockville Centre, the findings cannot be applied to other settings.

CHAPTER 4

DATA ANALYSIS AND FINDINGS

Introduction

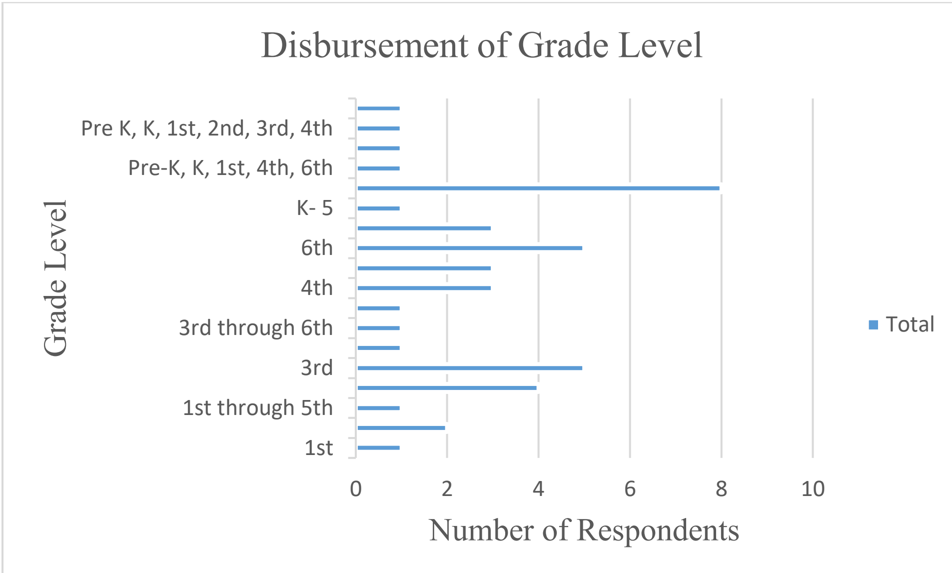
Mathematics Teaching Self- Efficacy and Instructional Practices Survey was adapted by the researcher from the Instructional Interactions Survey, with permission from Dr. Spillane of Northwestern University, School of Education and Social Policy (Appendix E). The wording of items taken from the original survey was not changed. The survey was shortened from the survey used by Dr. Spillane to gather data specific to the limits of this survey. Therefore, it contains three sections. The first section contains 19 items which are designed to gather data about mathematics teaching efficacy beliefs. The second section contains 11 items regarding mathematics instructional practices. The third section contains 18 items reflecting mathematics instructional beliefs (Appendix F).

The survey was electronically distributed to 47 elementary schools in the Diocese of Rockville Centre, on Long Island, NY. 93 surveys were completed, 90 females and 3 males. The disbursement of grade level can be found in Figure 2.

The data was downloaded from Google forms to SPSS. The data was used to answer the following research questions:

1. What are self- efficacy beliefs of teachers of elementary mathematics?
2. What instructional strategies characterize those of elementary mathematics teachers?
3. What are teachers' beliefs about student learning and mathematics instruction?
4. What is the relationship between reflection in action, classroom discourse, and teacher self- efficacy in mathematics instruction?

Figure 2. *Disbursement mathematics grade level taught by of respondents.*



Findings for Research Question 1:

The items contained in the first section of the Mathematics Efficacy and Instructional Practices Survey are originally adapted from the Science Teaching Efficacy Beliefs Survey (Riggs & Enochs, 1990). The items reflect two aspects which guide behavior, personal teaching efficacy beliefs, and outcome expectancy (Bandura, 1977; Riggs & Enochs, 1990). Therefore, to answer the first research question, the items were kept as separate constructs (Dellinger, et al. 2007; Enochs, Smith & Huinker, 2000; Riggs & Enoch, 1990). The items that comprise the scale that measures efficacy beliefs are shown in Table 1. The items designed to measure outcome expectancy are shown in Table 2.

Table 1 *Mathematics Teaching Self-Efficacy Scale*

Item	Description
E2	Teacher's ability to use manipulatives to explain mathematics
E4	Teacher's ability to answer student questions
E5	Teacher welcomes student questions
E6	Teacher's understanding of mathematical concepts
E11	Teacher welcomes principal observation of mathematics lesson
E13	Teacher's ability to explain concepts
E15	Teacher’s ability to get students interested in mathematics
E16	Teacher's ability to motivate difficult students
E17	Teacher's ability to increase student retention
E18	Teacher's ability to redirect difficult students
E19	Teacher's ability to respond to students' needs

Note. “E” identifies the item as a measurement for self-efficacy in first section of the survey.

Table 2 *Outcome Expectancy Scale*

Item	Description
OE1	Teacher effort
OE3	Student achievement and ineffective teaching
OE7	Teacher responsibility for student achievement
OE8	Student achievement and effective Teaching
OE9	Student Interest as related to teacher performance
OE10	Student Improvement and Teacher effectiveness
OE12	Teacher's ability to overcome student's inadequate math background
OE14	Student improvement due to teacher attention

Note. “OE” identifies the item as a measurement for outcome expectancy in the first section of the survey.

Several items were reversed scored to keep consistent values between negatively and positively worded items. These are shown below in Table 3.

Table 3 *Reverse Scored Items: Mathematics Efficacy and Outcome Expectancy*

Item	Description
E2	Teacher's ability to use manipulatives to explain mathematics
OE3	Student achievement and ineffective teaching
OE7	Teacher responsibility for student achievement
E11	Teacher welcomes principal observation of mathematics lesson
E13	Teacher's ability to explain concepts
E15	Teacher's ability to get students interested in mathematics

Note. Reversed scored items are recoded in SPSS as 5=1, 4=2, 2=4, 1=5

The Cronbach's alphas for the 11 mathematics self- efficacy items and the 8 outcome expectancy items were .79 and .72 respectively, indicating internal consistency of items (Lance, Butts, & Michels, 2006; Muijs, 2011; Tavakol & Dennick, 2011).

Although both scales contain valuable information about teachers' beliefs regarding the effectiveness of their teaching, only the Mathematics teaching self- efficacy (MTSE) scale was used to answer the question "*What are self- efficacy beliefs of teachers of elementary mathematics?*" As stated by Dellinger, et al. (2007, p. 752) "teacher self-efficacy beliefs can be defined as a *teacher's individual beliefs in their capabilities to perform specific teaching tasks at a specified level of quality in a specified situation",* and should not be combined in a score using outcome expectancy items, because these reflective of student performance as an outcome of many "teaching behaviors and learning behaviors of students" (p. 753), some of which may not be under the control of the teacher.

Therefore, the items specified in Table 1, *Mathematics teaching self-efficacy scale* were used by the researcher to create a variable Mathematics teaching self-efficacy score.

The sum of the items was calculated in SPSS. As explained in Table 3, several items were reverse scored to ensure consistent directionality. The researcher noted that there were 5 responses among the 465 used to compute the composite score. The missing data was replaced with the mean of the series score. The information is detailed in Table 4.

Table 4 *Missing Data Replaced with Series Mean: MTSES*

Respondent	Missing Item	Series Mean
11	E4	4.4
17	E15	4.2
57	E16	3.7
76	E6	4.5
79	E15	4.2

The researcher used this data to create two variables Mathematics Teaching Self-Efficacy Score Adjusted (MTSES_Adj) and Mathematics Teaching Self-Efficacy Adjusted Mean (MTSES_AdjM). These two scores were calculated for each respondent. The descriptive statistics for each variable are detailed in Table 5.

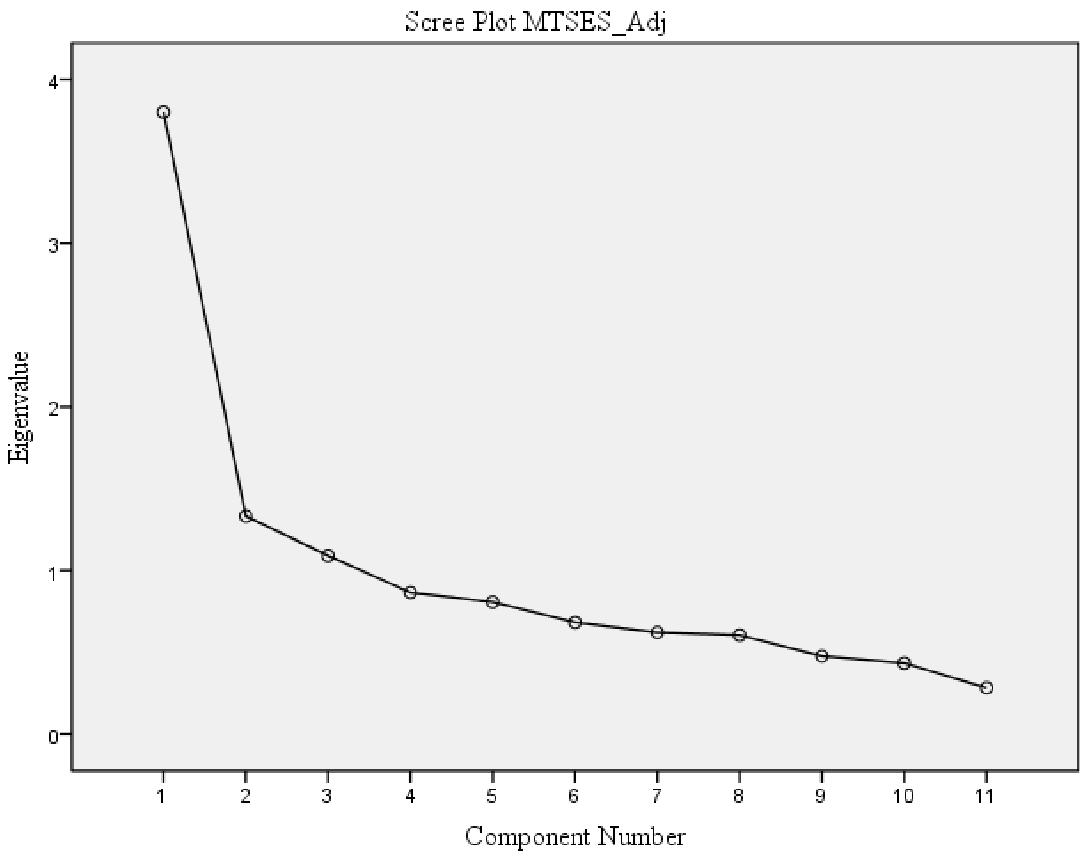
Table 5 *Descriptive Statistics for Mathematics Self- Efficacy Variables*

Variable	*M*	*SD*
MTSES_Adj	46.05	5.04
MTSES_AdjM	4.19	0.46

As shown in Table 5, the mean of the MTSES_Adj is 46.05 which is high. A closer look at the data shows that the standard deviation is 5.04, the variance is 25.40 and the range is 23. The mean of the MTSES_AdjM is 4.19, the standard deviation is .46 and the variance is .21. The range for this score is 2.09. This shows that although the mean of the scores for teacher self- efficacy beliefs is high, the variance is also high, as shown in the variance and range of the composite score, MTSES_Adj. Further investigation of the data will explore the cause of the variance and range of scores.

To do this, the researcher conducted principle components analysis to explain the variability of the MTSES_Adj, the teachers' self- efficacy beliefs. Principle components analysis is a data reduction technique which creates factors that will allow the researcher to interpret the larger series of data in a smaller number of components, to explain as much variance in teachers' self- efficacy as possible. The factors will then be examined and retained based on their eigenvalues, which is the variance extracted by a factor (Muijs, 2011; Salkind, 2014).

Figure 3 *Scree Plot for MTSES_Adj*



There are 11 factors extracted here, but most of them explain little variance. The plot does show a sharp drop in variance after the fourth factor. In further factor loading, the researcher extracted 4 factors which explained 64% of the variance in the MTSES_Adj. The researcher examined the pattern matrix to which indicates the importance of that variable to each factor. A second extraction was done of three factors. After examining internal consistency of these three components, the researcher did a third and final extraction (Appendix G).

Using the pattern matrix for two factors the researcher looked for relationships greater than .3 and less than .3. (Appendix G). It appears that rather than there being 11 separate math factors, as originally hypothesized, there are two factors which combine items. The scree plot in Figure 3 shows a decline in variance after the second factor. These combined factors are labeled in accordance with the researcher's interpretation of the variable loading. 10 items were used. One variable was excluded, E2, "*Teacher's ability to explain manipulatives*". The relationship to either component 1 or 2 was very small. (Appendix G)

Table 6 *Principle Components of MTSES_Adj*

Component	1	2
Name	Clarification and Communication	Accommodating student needs
Items	E4, E5, E6, E11, E13	E15, E16, E17, E18, E19
ά	0.76	0.69
Eigenvalue	3.8	1.33

Note. Descriptions of items are detailed in Table 1.

The five variables that loaded onto component or factor 1 were the survey items "*Teacher's ability to answer questions", "Teacher's understanding of math concepts", "Teacher's ability to explain concepts*", "*Teacher welcomes student questions*", and "*Teacher welcomes principal's observation*". These responses reflect the respondent's ability to communicate feedback to students. It was therefore labeled "Communication and Clarification". This component accounted for 34.56% of the variance in the teacher's mathematics self-efficacy score (MTSES_Adj). The Cronbach's alpha of .76 shows that it is an internally consistent factor.

The five variables that loaded onto component 2 were the survey items "*Teacher's ability to get students interested",* "*Teacher's ability to motivate students",* "*Teacher's* *ability to redirect students",* and "*Teacher's ability* to *respond to student needs*". These items reflect the teacher's ability to accommodate student needs and therefore was labeled as such. This component accounted for 12.11% of the variance in the MTSES_Adj. The Cronbach's alpha for this factor was .69.

These components explain the variance in the mathematics teaching self- efficacy beliefs of the respondents. It is notable that the teacher's belief in their ability to provide feedback and clarification to their students accounted for over 30% of the variance in efficacy beliefs of respondents. While components 1 and 2 are moderately correlated (r=.39), the effect of component 1on efficacy beliefs is almost three times that of component 2.

The principle components analysis detailed above established that there are two separate factors, and Cronbach's alpha has shown that these items form two internally consistent scales. The researcher added these items to make two new scales: Clarification and Communication Scale (CCS), and Accommodating Student Needs Scale (ASNS). The descriptive statistics for these two variables can be found in Table 7.

Table 7 *Descriptive Statistics: CCS and ASNS*

	M	*SD*	Variance
ASNS	19.65	2.54	6.54
ASN_M	3.93	.51	.26
CCS	22.14	2.86	8.18
CCS_M	4.43	.59	.35

Findings for Research Question 2

The second section of the Mathematics Efficacy and Instructional Practices Survey contains questions which were coded as measuring respondents' instructional practices (Appendix F). Once uploaded into SPSS, they were labeled as detailed in Table 8. The scoring of these items reflects the degree to which practices reflect the components of social constructivist instruction, as identified in Chapter 2. By reviewing the data, the researcher found 5 responses out of 1,173 in this series which were missing. The responses were replaced with the series mean (Table 9). As a result, the Cronbach's alpha was .67.

Table 8 *Mathematics Instructional Practices Scale*

Item	Description
P1	Students work individually without assistance from the teacher
P2	Students work individually with assistance from the teacher
P3	Students work together as a class with the teacher teaching the whole class
P4	Students work together as a class responding to one another
P5	Students work in small groups without assistance from each other
P6	Students work in small groups with assistance from each other
P7	Students explain the reasoning behind an idea
P8	Students represent and analyze relationships using graphs or tables
P9	Students work on problems for which there are no immediate methods of solution
P10	Students use computers to complete exercises or solve problems
P11	Students write equations to represent relationships

Table 9 *Missing Data Replaced with Series Mean: MIPS*

Respondent	Missing Item	Series Mean
21	P7	3.1
24	P5	2.3
43	P8	2.6
58	P5	2.3
86	P4	2.9

Table 10 *Descriptive Statistics MIP_Adj and MIP_AdjM*

	M	*SD*	Variance
MIP_Adj	29.63	4.03	16.20
MIP_AdjM	2.6	0.37	0.13

A high score reflects classroom practices in which students build concepts through reflection and discussion about experiences and define problems in context. A low score reflects instruction based on a behavioral perspective: designed to focus on small, discrete units of work and carefully designed tasks (Smith, 2011, pp. 7-8). A minimum of 17 was scored, showing a more teacher- centered classroom environment (Table 10). The variance of the MIP_Adj was 16.20.

The MIP_AdjM gave a mean score to each respondent. The mean was 2.69, which reflects a reliance by most respondents on traditional, teacher- centered practices.

The researcher conducted a principle components analysis of the items used in the MIP scale. Figure 4 shows the Scree Plot associated with the principle components analysis. The Scree Plot shows a leveling of variation after the fourth component. The researcher hypothesized that these components reflected the factors of discourse, inquiry, collaboration and creativity/ higher order thinking skills.

Figure 4 *Scree plot: Mathematics Instructional Practices*

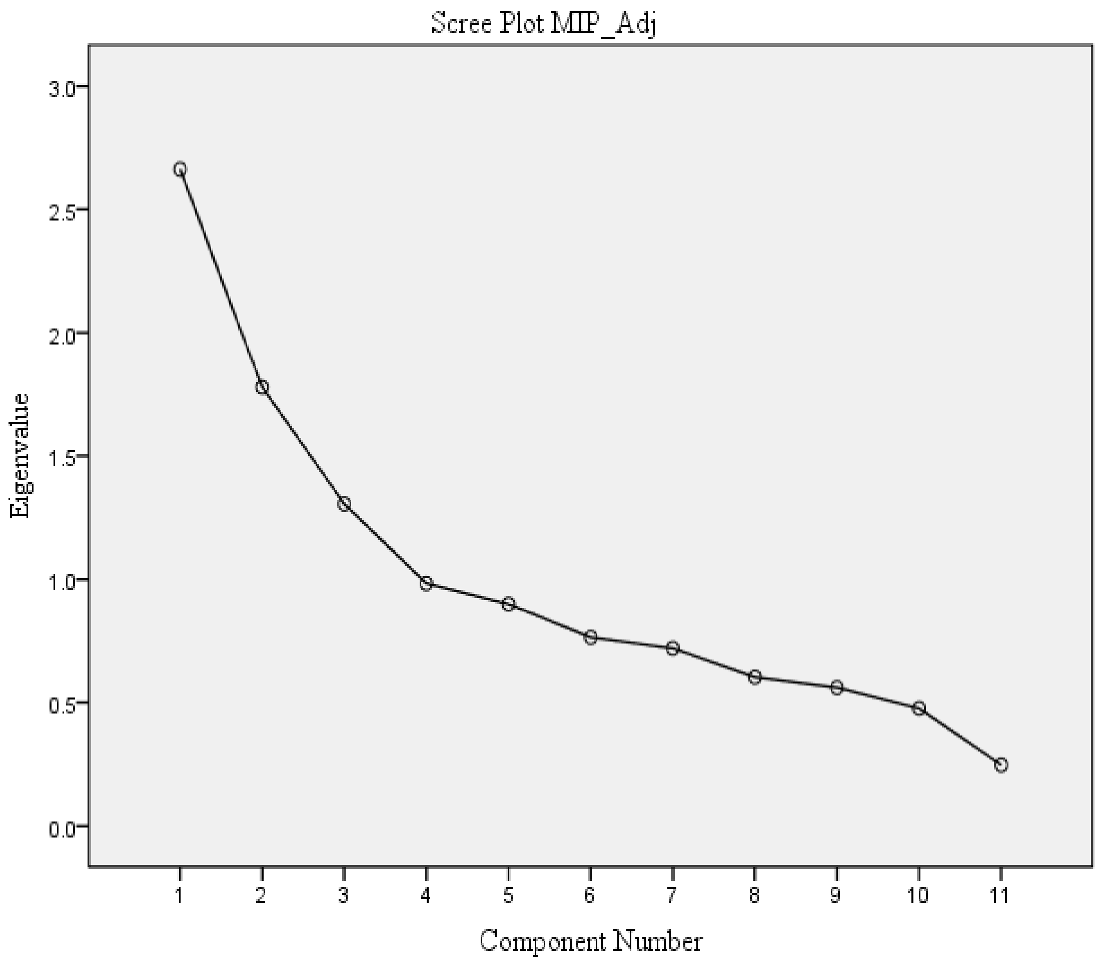


Table 11 *Principle Components of Mathematics Instructional Practices*

Component	1	2	3
Name	Creativity/ HOTS	Discussion	Collaboration and Inquiry
Items	P7, P8, P9, P10, P11	P1, P2, P3, P4	P5, P6, P8, P9, P10
ά	0.64	0.62	0.63
Eigenvalue	2.66	1.78	1.3

The items in Component 1 reflect the use of higher order thinking skills and creativity as part of the respondents' instructional practices. It represents the respondents' emphasis on mathematical reasoning and problem- solving. Component 2 is a measure of the communication style in the classroom. The items in Component 3 represent of collaboration and inquiry. Although there is some cross- loading, the items combine to create scales that represent underlying variables associated with respondents' instructional practices (Table 12). Creativity and higher order thinking skills (CHOTS) had a mean of 12.72 variance of 6.05 among the respondents. The mean score among the respondents as shown in CHOTS_M was 2.54 among respondents which reflects the average response between "some lessons" and "most lessons". Discussion and communication (DCS) had a mean of 11.75 and a variance of 4.11 among respondents. The overall mean score (DCS_M) of 2.94 reflects overall responses close to "most lessons". Collaborative learning strategies (CIP) had a mean of 12.07 and a variance of 6.18. The average score among all responses (CIP_M) was 2.41, which fell between "some lessons" and "most lessons", but was closer to "some lessons".

Table 12 *Descriptive Statistics: Principal Components of MIP*

	M	*SD*	Variance
CHOTS	12.72	2.46	6.05
DCS	11.75	2.03	4.11
CIP	12.07	2.49	6.18
CHOTS_M	2.54	0.49	0.24
DCS_M	2.94	0.51	0.26
CIP_M	2.41	0.50	0.25

Note. A lower score reflects student tasks based on traditional model.

Findings for Research Question 3

To assess the respondents' beliefs about student learning and mathematics instruction, examined the responses to the third section of the survey, the Mathematical Beliefs Inventory (Appendix F). This was originally adapted from the Fennema- Sherman Short Form. Once uploaded into SPSS they were coded as shown in Table 13. The responses reflect the beliefs that respondents hold regarding student learning in mathematics.

For the purposes of this study, a higher score indicated that respondents believed that mathematical learning should focus on the cognitive processes of students. A lower score indicated agreement that the use of direct approach was more effective (Hopkins & Spillane, 2013). Therefore, items needed to be reverse- scored to maintain directionality of responses. Several blank responses were replaced with the series mean (Table 15).

Table 13 *Mathematical Beliefs Inventory*

Item	Description
MB1	Encourage students to find their own solutions to problems even if they are inefficient.
MB2	Most students have to be shown how to solve simple math problems.
MB3	Recall of number facts should precede the development of an understanding of the related operation.
MB4	Students should master computational procedures before they are expected to understand how those procedures work.
MB5	Students need explicit instruction on how to solve word problems.
MB6	Teachers should allow students to argue out their own ways to solve simple word problems.
MB7	The goals of instruction in mathematics are best achieved when students find their own methods for solving problems.
MB8	Most students can figure out ways to solve many mathematical problems.
MB9	Time should be spent practicing computational procedures before are expected to understand procedures.
MB10	Students should not solve simple word problems until they have mastered some number facts.
MB11	Students attending to teacher explanations.
MB12	Students must be good listeners.
MB13	Teachers should model specific procedures for solving word problems.
MB14	Mathematics should be presented to children in such a way that they can discover relationships for themselves.
MB15	Students should understand computational procedures before they master them.
MB16	Time should be spent practicing computational procedures before students spend much time solving problems.
MB17	Students will not understand an operation until they have mastered some of the relevant number facts.
MB18	Teachers should allow students who are having difficulty solving a word problem to continue to try to find a solution.

Table 14 *Reverse Scored Items: Mathematical Belief Inventory*

Item	Description
MB2	Encourage students to find their own solutions to math problems even if they are inefficient
MB3	Recall of number facts should precede the development of an understanding of the related operation.
MB4	Students should master computational procedures before they are expected to understand how those procedures work.
MB5	Students need explicit instruction on how to solve word problems.
MB9	Time should be spent practicing computational procedures before students are expected to understand the procedures.
MB10	Students should not solve simple word problems until they have mastered some number facts.
MB11	Students attending to teacher explanations.
MB12	Students must be good listeners
MB13	Teachers should model specific procedures for solving word problems.
MB16	Time should be spent practicing computational procedures before students spend much time solving problems.
MB17	Students will not understand an operation until they have mastered some of the relevant number facts.

Note. Reversed scored items are recoded in SPSS as 5=1, 4=2, 2=4, 1=5

Table 15 *Missing Data Replaced with Series Mean*

Respondent	Missing Item	Series Mean
10	MB3	2.5
42	MB4	3
57	MB7	3.5
57	MB9	2.8
59	MB15	4
64	MB4	3
82	MB16	2.4

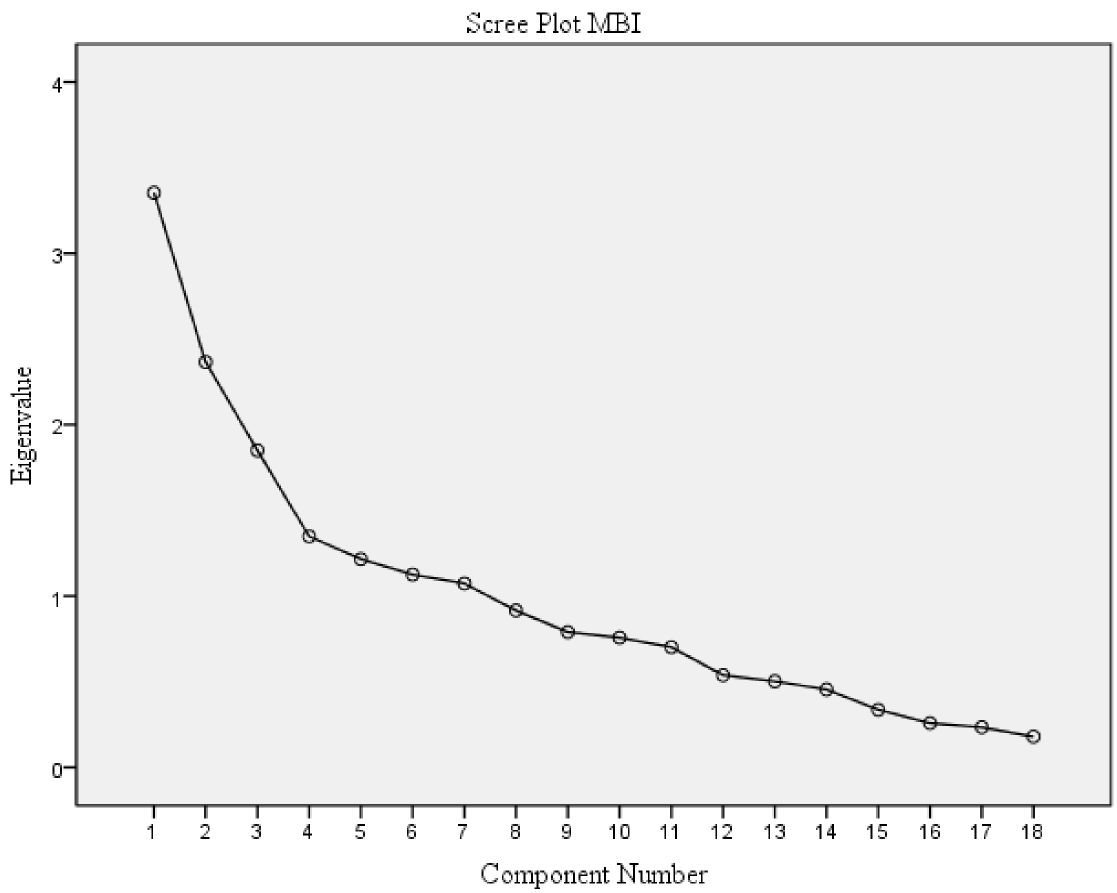
This produced a Mathematical Belief Inventory Scale (Cronbach's alpha= .70). This scale was used to create two variables, MBI_Adj and MBI_AdjM. Table 16 contains the descriptive statistics for these variables.

Table 16 *Descriptive Statistics for Mathematical Beliefs Inventory*

	M	*SD*	Variance
MBI_Adj	54.81	7.08	50.08
MBI_AdjM	3.04	.39	.15

A factor analysis was also conducted to find how the items in the inventory are associated with each other. The 18 items load onto three components, or factors, which account for 42% of the variance in respondents' beliefs (Appendix J). Five items were identified as part of a subscale with an eigenvalue of 3.35, (Cronbach's alpha= .70). The items are MB2, MB3, MB4, MB9, MB17. The subscale items describe respondents' beliefs about the necessity of practicing procedures prior to understanding them. A high score on these items reflects the "teacher's conception of mathematics… as that of a dynamic subject rather than a fixed body of knowledge" (Ross, McDougall, Hogaboam-Gray & LeSage, 2003, p. 348).

Figure 5 *Scree Plot of Mathematical Beliefs Inventory*



This underlying component of mathematical beliefs was used to create a new scale. By removing MB2, the Cronbach's alpha was .76. The new variable is labeled Mathematical Belief Conceptual Understanding (MBCU). The mean of this scale is labeled MBCU_M (Table 17).

Table 17 *Descriptive Statistics Mathematical Beliefs of Conceptual Understanding*

	M	*SD*	Variance
MBCU	11.07	3.27	10.71
MBCU_M	2.77	.82	.67

The scoring of this scale demonstrates the respondents toward a belief that students must practice mathematics before they understand underlying concepts, a more traditional belief in mathematics instruction.

Findings for Research Question 4

Research question 4 asked "What is the relationship between reflection in action, classroom discourse, and teacher self- efficacy in mathematics?"

To answer this question, the researcher conducted Pearson coefficients between variables and examined the meaning of the Pearson correlation as an effect size statistic (R^2).

Table 18 describes the variables used to account for the variance within mathematics teaching self- efficacy.

A correlation coefficient was computed between the two self- efficacy variables. The results of the correlational analysis presented in Table 19 show that the correlation was statistically significant ($p<.01$) and equal to .46. This indicates a moderate, direct relationship between the two variables (Salkind, 2014, p.93). In general, if teachers feel that they can provide feedback to students, they also feel that they are able to accommodate individual student needs.

Table 18 *Variables in MTSES and Definitions*

Variable	Definitions
Clarification and Communication	High scores on this variable indicate that the respondents have positive self-efficacy beliefs in providing feedback to students
Accommodating Student Needs	High scores on this variable indicate that the respondents have positive self-efficacy beliefs in accommodating individual student needs.

Table 19 *Correlation between CCS and ASNS*

		CCS
ASNS	*r*	0.46
	P	.00
	R^2	21%

Table 20 *Variables in Mathematics Instructional Practices and Definition*

Variable	Definitions
Creativity and Higher Order Thinking Skills	High scores on this variable indicate the use of complex, open-ended problems, and student learning through discovery.
Discussion and Communication	High scores on this variable indicate the use of discussion to facilitate learning.
Collaboration	High scores on this variable indicate the use of student interaction to promote learning.

Correlation coefficients were computed among the three mathematical instructional practices variables. The results of the correlational analysis presented in Table 21 show that two of the three correlations were significant ($p < .01$), therefore, the null hypothesis is rejected. The results show a positive low relationship (Salkind, 2014, p. 93) between the use of discussion in the classroom and creativity and higher order thinking skills ($r_{CHOTS-DCS} = .28$). In terms of percentage of variance, 8% in the use of creativity and higher order thinking skills can be explained using discussion in the classroom. The correlation between the use of creativity/higher order thinking skills and collaboration is .76, indicating a strong, positive direct relationship between the two practices. The relationship between collaboration and discussion was not statistically significant. In general, a teacher who uses complex, open- ended problems will most likely use collaboration, and possibly use discussion to facilitate learning.

Table 21 *Correlation among components of Mathematics Instructional Practice Scale*

		DCS	CIP
CHOTS	*r*	0.28*	0.76*
	P	0.01	0.00
	R^2	8%	58%
CIP	*r*	0.17	
	P	0.10	

A correlation coefficient was computed between mathematical beliefs and mathematics self- efficacy (MBI and MTSES_S). The results of the analysis presented on Table 22 that the relationship was statistically significant ($p = .03$). Therefore, the null hypothesis was rejected. The correlation was .25, which indicates a positive, low relationship ($R^2_{MBI-MTSES} = 6\%$) between teacher's beliefs about student learning and their mathematics teaching self- efficacy beliefs. In general, if teachers have high scores in their mathematics teaching self- efficacy, they may also have high scores in the mathematics belief inventory, reflecting the belief that mathematics is best taught using student centered- learning, although the relationship is weak (Table 22).

Table 22 *Correlations: Mathematics -Self Efficacy, Mathematical Practices and Mathematical Beliefs about Student Learning*

		MTES	MBI
	r		0.25*
	P		.02
MTES	R^2		6%
	r	.05	0.25*
	P	.64	.02
MIP	R^2		6%

**Note.* Correlation is significant at the 0.05 level (2-tailed).

A correlation coefficient was also computed between mathematics instructional practices (MIP) and mathematics beliefs (MBI). The results of the analysis show that the relationship was statistically significant (p=.02, $r_{MIP\text{-}MBI}$=.25). In general, if teachers have a high score in the mathematics beliefs inventory, they may also use student-centered learning, although the relationship is weak (Table 22).

Table 23 *Correlations: Mathematical Beliefs and Subscale Variables associated with Mathematics Instructional Practices*

		MBI
CHOTS	*r*	.26
	p	.02
	R^2	7%

**Note*. Correlation is significant at the 0.05 level (2-tailed).

A correlation coefficient was computed among mathematical beliefs and the three variables identified within mathematics instructional practices: creativity/higher order thinking practices (CHOTS), discussion and communication (DCS), and collaborative learning practices (CIP). The results, presented on Table 23, show that only one correlation was statistically significant.

The correlation between mathematical beliefs and the use of creativity/ higher order thinking skills was statistically significant (p=.02). The results show a low, positive relationship ($r_{MBI\text{-}CHOTS}$=.26, $R^2_{MBI\text{-}CHOTS}$= 7%). In general, the results suggest that if a teacher has a high score on the mathematics beliefs inventory (suggesting agreement that mathematics is best taught using inquiry- based learning), they may use complex, open-ended problems and student discovery, but may not necessarily use discussion or collaboration in the classroom.

Table 24 *Correlations between mathematical beliefs clarification and accommodating student needs*

		MBI
MBI	r	1
	p	
ASNS	r	.12
	p	.27
CCS	r	.25*
	p	.02
	R^2	6%

**Note*. Correlation is significant at the 0.05 level (2-tailed).

Table 24 shows the results of a correlational analysis between mathematical beliefs and the teachers perceived ability to provide clarification and feedback (CCS) and accommodate individual student needs.

There was no statistically significant relationship between the teacher's ability to accommodate individual student needs, but there was a statistically significant relationship between mathematical beliefs and perceived ability to provide clarification and feedback to students (p= .02). The results indicate a low, positive relationship ($r_{ccs-MBI}$= .25, $R^2_{CCS-MBI}$= 6%). The results suggest that if a teacher scores high on the mathematics inventory, they may also score high on their perceived ability to provide feedback to students.

CHAPTER 5

DISCUSSION, SUMMARY AND RECOMMENDATIONS

Discussion

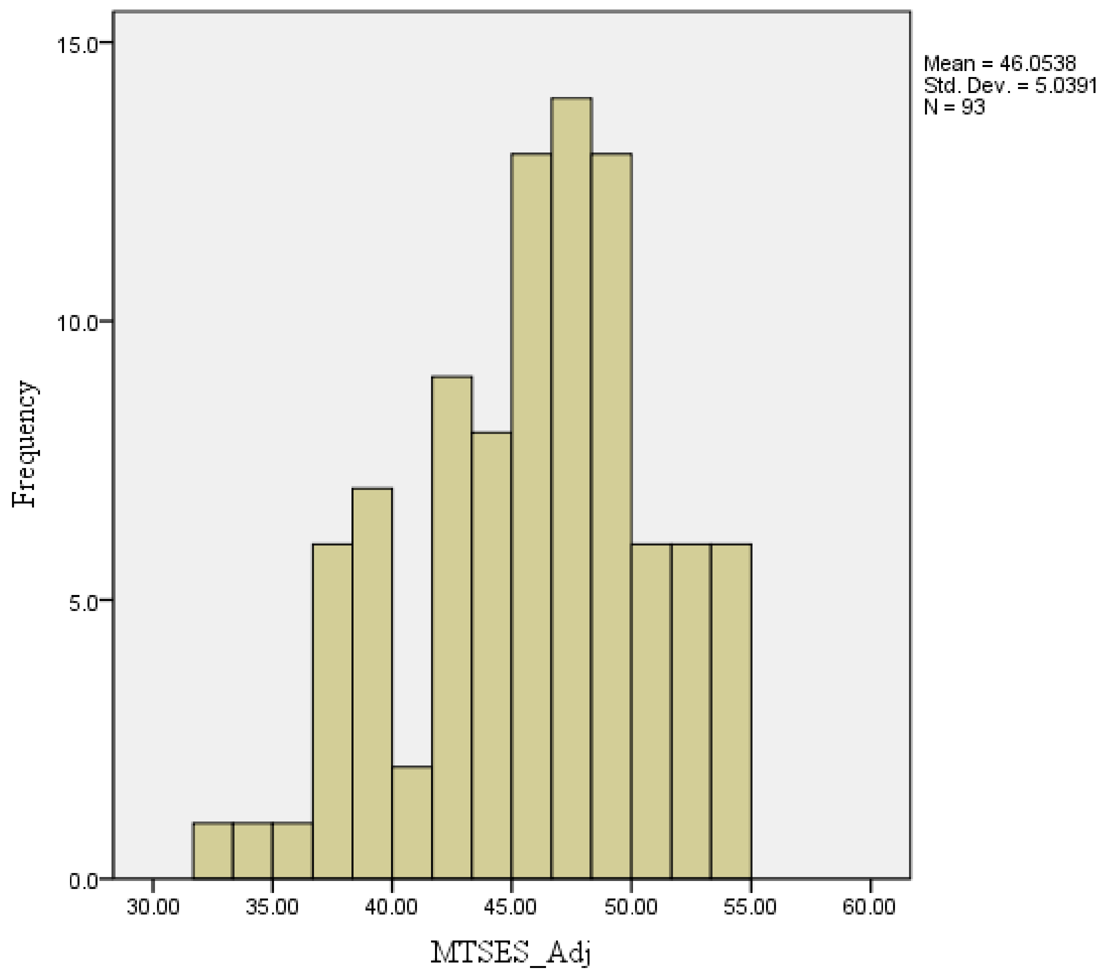
The goal of this study was to explore the effect of instructional strategies on self-efficacy beliefs to more fully understand the relationship between the two groups, and find specifically what factors of instruction, if any, improve mathematics teaching self-efficacy beliefs of teachers.

Research Question 1

The first research question of this study asked, "*What are self- efficacy beliefs of teachers of elementary mathematics?"*. To answer that question, the researcher analyzed 11items from the first section of the Mathematics Efficacy and Instructional Practices Survey. These 11 items reflect the respondents' beliefs in their ability to perform specific mathematical teaching tasks at a specific level of performance in their specific teaching situation (Dellinger, et al., 2007).

The results found in Table 5 show a mean score of 46.05, which reflects an overall high score for mathematics self- efficacy beliefs for the respondents of this survey. The standard deviation is 5.04, and the variance is 25.40. This high standard deviation and variance indicate that the data is spread far from the mean score, 46.05. In fact, the range for this data set was 23 points: the maximum score was 55, and the minimum score was 32. Figure 6 is a histogram which illustrates the dispersion of the respondents' total mathematics self- efficacy score.

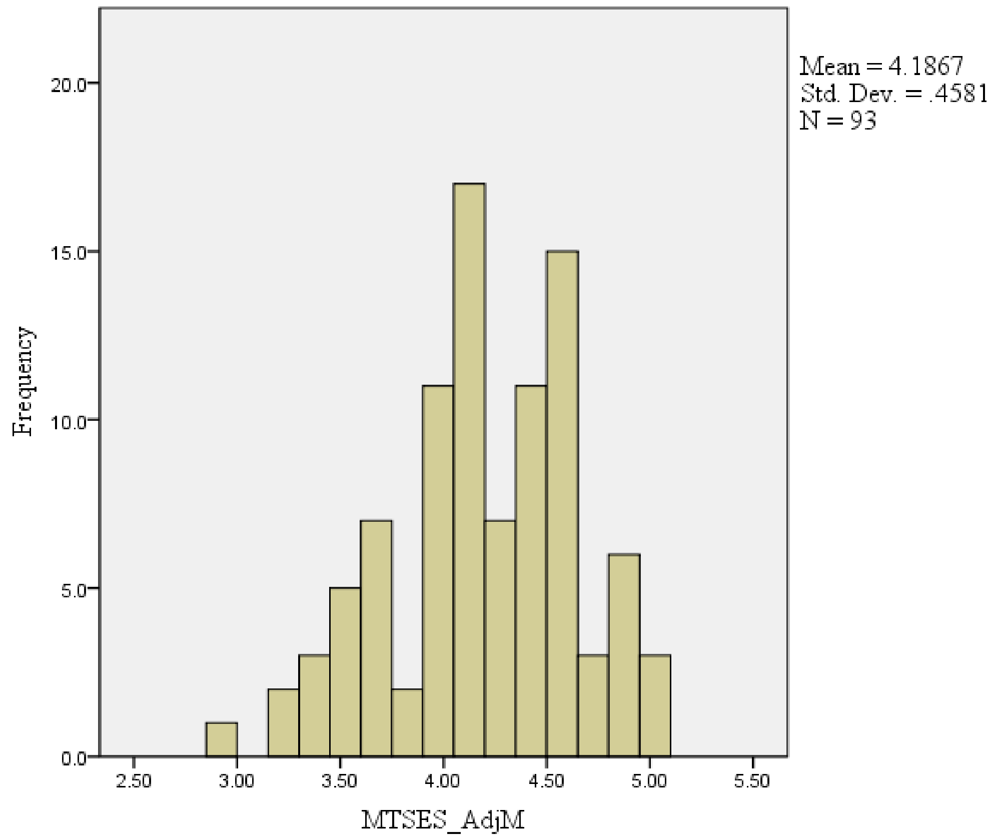
Figure 6 *Histogram: Dispersion of Total Mathematics Self- Efficacy Score*



The researcher also assigned a mean score to each respondent, which was their total score, divided by the number of items (11). The mean for this among the respondents was 4.19, which is also high. The standard deviation was .46, and the variance was .21. Although these numbers are not high, it is important to note that the range for this score was 2.09. The maximum score was 5.0 and the minimum score was

2.91, which was rather low. This data shows that although the mean of the scores for teachers' self- efficacy beliefs was high, the range of scores was wide. Figure 7 is a histogram of scores for the mathematics self- efficacy mean scores.

Figure 7 *Histogram: Dispersion of Mean Mathematics Self- Efficacy Scores*



The researcher then conducted a principle components analysis of the Mathematics Self- Efficacy score. Two subcomponents were extracted: Clarification and Communication, and Accommodating Student Needs. These components explain the variance in the mathematics teaching self- efficacy beliefs of respondents. As noted in Chapter 4, respondents' belief in their ability to provide feedback and clarification to students accounted for 34.55% of the variance of the mathematics self- efficacy score, and almost three times more than that of their belief in their ability to accommodate individual student needs (Appendix G). The range for the CCS was 11 with a maximum score for CCS was 25, the minimum score was 14. The variance was 8.16, which is high. A closer look at this scale will further explain areas in which respondents have low self-efficacy in mathematics teaching.

For the item E4, *"Ability to answer student questions"*, the mean response was 4.42, which corresponds with "agree/ strongly agree". In fact, 89.2% of the responses fell under "agree/ strongly agree". The range of responses was 4 (maximum= 5, minimum= 1). The standard deviation was .80 and the variance was .64. The table below shows the frequencies of the responses.

Table 25 *Frequency of Responses: "Teacher's ability to answer student questions"*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Strongly Disagree	1	1.1	1.1	1.1
	Disagree	2	2.2	2.2	3.2
	Neutral	6	6.5	6.5	9.7
	Agree	31	33.3	33.3	43.0
	4.4	1	1.1	1.1	44.1
	Strongly Agree	52	55.9	55.9	100.0
	Total	93	100.0	100.0	

The table above indicates that 9 responses were either negative or neutral in teacher's belief in their ability to answer student questions. The responses "strongly disagree/ disagree/ neutral" accounted for 9.7% of the responses. (This reflects one response left blank and replaced with series mean, 4.4.)

Table 26 *Frequency of Responses:* "*Welcomes student questions"*.

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Disagree	1	1.1	1.1	1.1
	Neutral	4	4.3	4.3	5.4
	Agree	14	15.1	15.1	20.4
	Strongly Agree	74	79.6	79.6	100.0
	Total	93	100.0	100.0	

The table above shows the frequencies of responses to the item E5, *"Welcomes student questions"*. The mean for this response was 4.73 (*SD*= .59*,* variance= .35). As shown above only 5.4% of responses were "disagree" or "neutral"). No responses were left blank.

In comparing the responses for teachers' belief in their ability to answer student questions and the responses for welcoming student questions, we can conclude that although teachers may welcome student questions, they do not always feel confident in their ability to answer these questions sufficiently.

To more fully understand this dynamic, the researcher examined the frequency for the response E6, *"Teacher's understanding of mathematical concepts"* (*M*=4.48, *SD*= .7). The data found on Table 27 shows that 87.1% of respondents feel confident in their understanding of mathematical concepts, although 11.8% feel neutral about their understanding of mathematical concepts. This can be interpreted as an area in which respondents may need improvement. Table 28 shows that 9.7% of responses to E13, *"Teacher's ability to explain concepts"* were "disagree/ neutral (*M*=4.34, *SD*= .72). The frequency of responses to E11, *"Welcomes principal observation"* are found on Table 29.

The mean for this response was 4.16, and the standard deviation was 1.11. (Table 29), much higher than the other items which make up the Communication and Clarification Scale (CCS). Table 30 shows that 9.7% of respondents did not welcome principal observation, and 11.8% of responses were neutral. Therefore, 21.5% of respondents indicated a lack of confidence in their performance in front of their superiors.

Table 27 *Frequency of Responses: "Teacher's understanding of mathematical concepts"*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Neutral	11	11.8	11.8	11.8
	Agree	26	28.0	28.0	39.8
	4.5	1	1.1	1.1	40.9
	Strongly Agree	55	59.1	59.1	100.0
	Total	93	100.0	100.0	

Table 28 *Frequency of Responses: "Teacher's ability to explain concepts"*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Disagree	2	2.2	2.2	2.2
	Neutral	7	7.5	7.5	9.7
	Agree	41	44.1	44.1	53.8
	Strongly Agree	43	46.2	46.2	100.0
	Total	93	100.0	100.0	

Table 29 *Frequency of Responses: "Welcomes principal observation"*

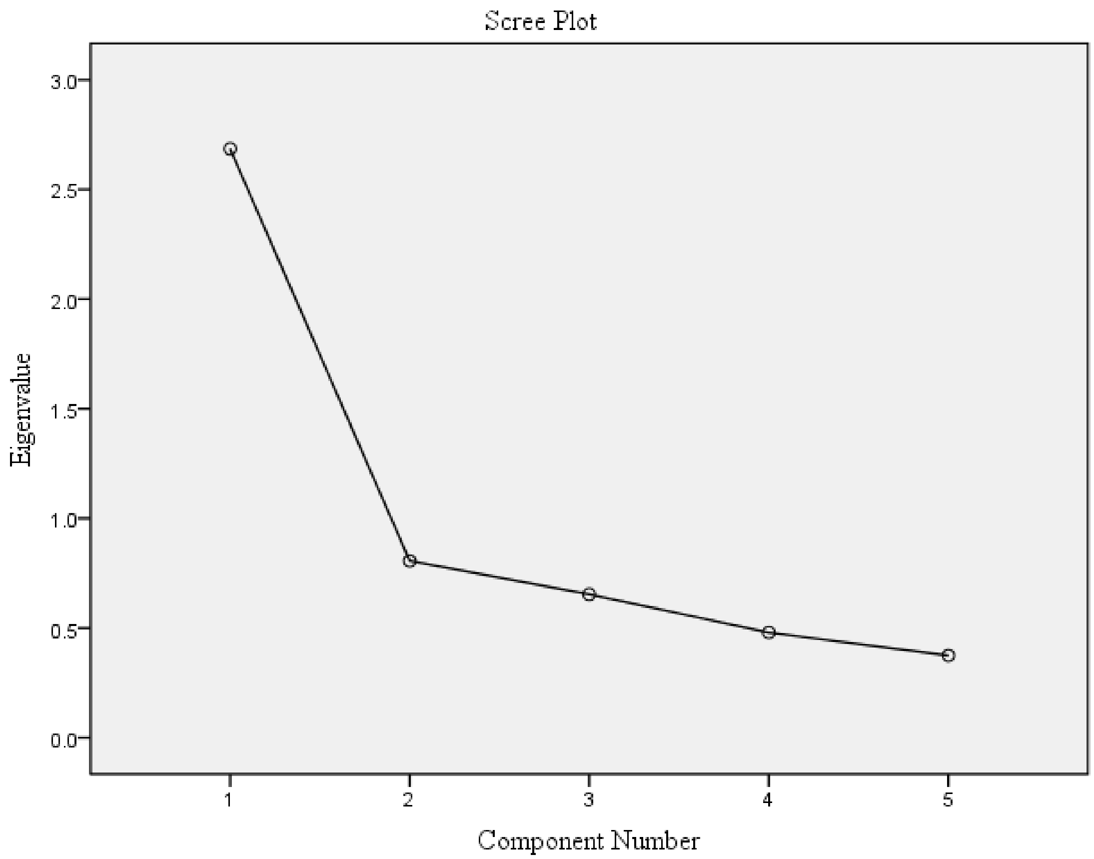
		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Strongly Disagree	4	4.3	4.3	4.3
	Disagree	5	5.4	5.4	9.7
	Neutral	11	11.8	11.8	21.5
	Agree	25	26.9	26.9	48.4
	Strongly Agree	48	51.6	51.6	100.0
	Total	93	100.0	100.0	

Table 30 *Descriptive Statistics: Components within the Clarification and Communication Scale*

	M	*SD*	*Variance*
Ability to Answer Student Questions	4.42	0.80	.64
Welcome Student Questions	4.73	0.59	.35
Understanding of Mathematical Concepts	4.48	0.70	.49
Teacher's ability to explain concepts	4.34	0.71	.51
Welcome principal observation	4.16	1.11	1.22

The researcher conducted a principle components analysis for this scale to better understand this high variance. The scree plot below (Figure 8) indicates that two of the five components account for 69.84% variance in the Clarification and Communication Scale.

Figure 8 *Scree Plot of Clarification and Communication Scale*



The principal component analysis revealed that the following items accounted for 57.71% of the variance in the Clarification Scale: E4, E5, and E6. The eigenvalue for these items was 2.69. As stated in Chapter 4 (Table 20), the Clarification and Communication Scale reflects the teachers' self- efficacy beliefs in providing feedback to students.

Table 31 *Pattern Matrix: Components of the Clarification and Communication Scale*

	Component	
	1	2
Ability to Answer Student Questions	.83	-.07
Welcome Student Questions	.82	-.03
Understanding of Mathematical Concepts	.67	.23
Welcome principal observation	-.10	.96
Teacher's ability to explain concepts	.29	.67

Note. Extraction Method: Principal Component Analysis. Rotation Method: Oblimin with Kaiser Normalization.

a. Rotation converged in 5 iterations.

The variance explained in Table 31 indicates that the teachers' perceived understanding of concepts, their confidence in their ability to answer student questions and welcoming student questions are the main reasons teachers may not feel confident in mathematics teaching. Correlation coefficients were computed among the five components of the Clarification and Communication Scale. The results of the correlational analysis presented in Table 33 show that the correlations were significant. Most notably, the respondents' understanding of mathematical concepts had a moderate, direct relationship with all the other four components of the CCS.

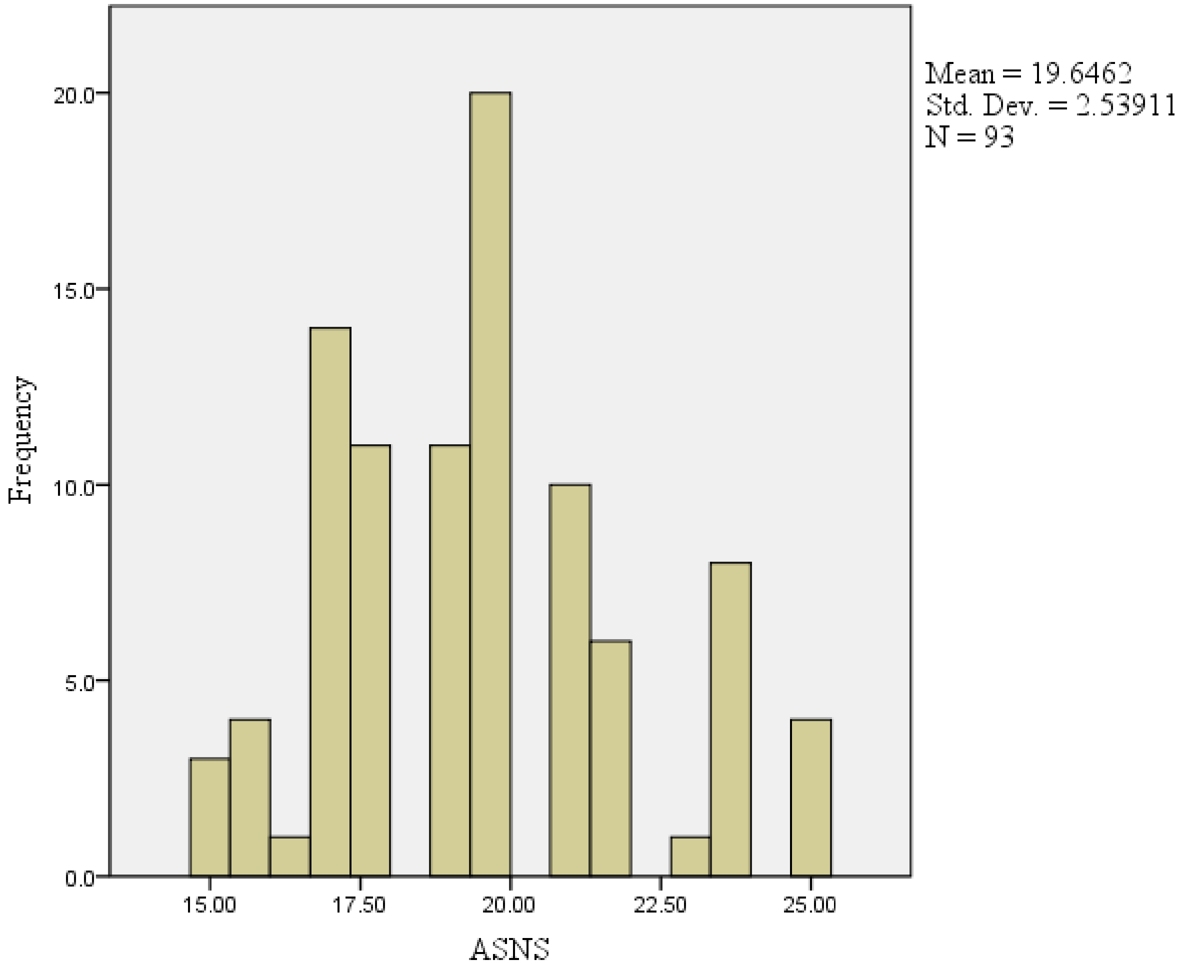
Table 32 *Correlations among Components of the Clarification and Communication Scale*

		Welcome principal observation	Teacher's ability to explain concepts	Ability to Answer Student Questions	Welcome Student Questions
Welcome principal observation	*r*	1	.52**	.32**	.25*
	p		.000	.00	.02
	R^2		27%	10%	6%
Teacher's ability to explain concepts	*r*	.52**	1	.34**	.50**
	p	.00		.00	.00
	R^2	27%		12%	25%
Ability to Answer Student Questions	*r*	.32**	.34**	1	.42**
	p	.00	.00		.00
	R^2	10%	12%		18%
Welcome Student Questions	*r*	.25*	.50**	.42**	1
	p	.02	.000	.00	
	R^2	6%	25%	18%	
Understanding of Mathematical Concepts	*r*	.40**	.48**	.49**	.47**
	p	.00	.00	.00	.00
	R^2	16%	23%	24%	22%

*Note*** Correlation is significant at the 0.01 level (2-tailed).
* Correlation is significant at the 0.05 level (2-tailed).

The ability to explain concepts is related to the teacher's confidence in principal observation. As shown in both Table 31 and Table 32. These components influence on the respondents' self- efficacy beliefs in their ability to provide feedback to students. The eigenvalue for this second component was not very high (.81) indicating that it had less influence on the self- efficacy beliefs of teachers.

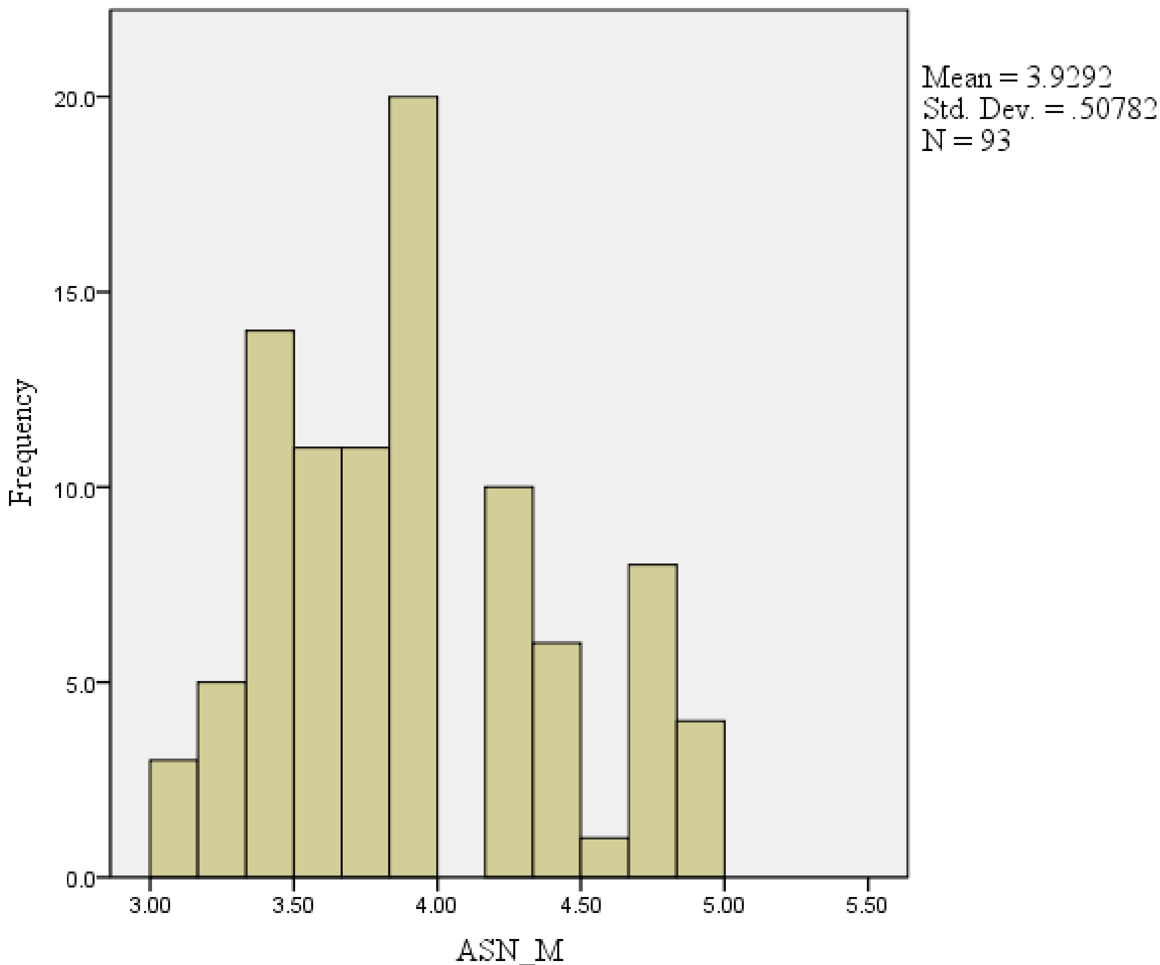
Figure 9 *Histogram: Dispersion of Total Score, Accommodating Student Needs*



The second component which made up the total mathematics teaching self-efficacy score minimum (MTSES) was the teachers' beliefs in their ability to accommodate student needs. This subcomponent of the MTSES was labeled "Accommodating Student Needs Scale" (ASNS). This factor accounted for 12.11% of the variance in the MTSES (Appendix G). The histogram in Figure 9 shows the dispersion of responses (*M*=19.66, *SD*=2.54). The variance was 6.46. The range was 10 points (maximum= 25, = 15).

The researcher also explored the dispersion of the mean score for accommodating student needs (*M*=3.93, *SD*=.51). The variance for this score was .26, and the range was 2 points.

Figure 10 *Histogram: Dispersion of Mean Score, Accommodating Student Needs*



As noted in Table 18, high sores on his variable indicate that the respondents have positive self-efficacy beliefs in accommodating individual student needs. To better understand the variance of the scale and the wide range of responses for the ASNS (total score), the researcher looked more closely at the items in this scale: E15, E16, E17, E18, and E19. The frequency for the responses to these items can be found below.

Table 33 *Frequency of Responses: "Respondents' ability to get students interested in mathematics"*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Disagree	3	3.2	3.2	3.2
	Neutral	11	11.8	11.8	15.1
	Agree	44	47.3	47.3	62.4
	4.2	2	2.2	2.2	64.5
	Strongly Agree	33	35.5	35.5	100.0
	Total	93	100.0	100.0	

The data shows that 3.2% of respondents felt negatively about their ability to get students interested in mathematics (E15). However, 11.8% of the respondents were neutral in their response. In addition, 2 responses (2.2%) were left blank. Although the researcher replaced this with the series mean, 4.2, we can also infer that perhaps as much as 14% of respondents felt uncertain about their ability to get students interested in mathematics. Conversely, at least 86% of respondents felt confident in their ability to get students interested in mathematics. This is a positive finding.

The frequency of responses for item E16, "*Respondents' ability to motivate difficult students*" is presented in Table 3. The data shows that 38 responses were neutral or given a negative response ("disagree). This means that 41% of respondents did not feel confident in their ability to motivate difficult students. One response was left blank and replaced with the series mean (3.7). The standard deviation for this response was .89

and therefore the variance was .79. The range for this response was 3 points. It can be inferred that this is an area in which teachers need more support.

Table 34 *Frequency of Responses: "Respondents' ability to motivate difficult students"*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Disagree	8	8.6	8.6	8.6
	Neutral	30	32.3	32.3	40.9
	3.7	1	1.1	1.1	41.9
	Agree	35	37.6	37.6	79.6
	Strongly Agree	19	20.4	20.4	100.0
	Total	93	100.0	100.0	

The responses to E17, "*Respondents' ability to increase student retention",* had a mean of 3.69 and a standard deviation of .77. The variance of .59 indicates that most responses tended toward the mean of "neutral". The data presented on Table 35 details the frequency of responses for this item.

Table 35 *Frequency of responses: "Respondents' ability to increase student retention"*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Strongly Disagree	1	1.1	1.1	1.1
	Disagree	4	4.3	4.3	5.4
	Neutral	28	30.1	30.1	35.5
	Agree	50	53.8	53.8	89.2
	Strongly Agree	10	10.8	10.8	100.0
	Total	93	100.0	100.0	

Although 33 responses (35.5%) were either negative or neutral, 60 responses were positive for this item. Although this is an area where teacher support is needed, the majority respondents felt confident in their ability to find ways to increase student retention of material

The responses for item E18, "*Respondents' ability to redirect direct difficult students"* had a mean of 4.17, (*SD*= .72, variance =. 51) This indicates that most respondents had confidence in their ability to redirect difficult student in the classroom A closer look at the data shows that the range was 3 points for this item. Table 36 shows that the low score of "disagree" was only given for 2.2% of the responses, and 11.8% were neutral. Therefore, 86% of responses were positive for this item.

Table 36 *Frequency of Responses: "Respondents' ability to redirect difficult students"*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Disagree	2	2.2	2.2	2.2
	Neutral	11	11.8	11.8	14.0
	Agree	49	52.7	52.7	66.7
	Strongly Agree	31	33.3	33.3	100.0
	Total	93	100.0	100.0	

Table 37 *Frequency of Responses: "Respondents' ability to respond to student needs"*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Disagree	1	1.1	1.1	1.1
	Neutral	26	28.0	28.0	29.0
	Agree	47	50.5	50.5	79.6
	Strongly Agree	19	20.4	20.4	100.0
	Total	93	100.0	100.0	

The responses for E19, "*Respondents' ability to respond to student needs",* (*M*=3.9, *SD*=.72) presented in Table 37 show that 28% of responses were neutral and 1.1% were negative. This indicates a lack of respondents' confidence their ability to respond to student needs.

Table 38 *Correlation among the Components of the Accommodating Student Needs Scale*

		Ability to get students interested in mathematics	Ability to increase student retention	Ability to redirect difficult students
Ability to get students interested in mathematics	*r*	1	.20	.25*
	p		.06	.02
	R^2			6%
Ability to increase student retention	*r*	.200	1	.26*
	p	.06		.01
	R^2			7%
Ability to redirect difficult students	*r*	.25*	0.26*	1
	p	.017	.01	
	R^2	6%	7%	
Ability to motivate difficult students	*r*	0.33**	0.29**	0.23*
	p	.00	.00	.03
	R^2	11%	8%	5%
Ability to respond to students' needs	*r*	.30**	0.34**	0.37**
	p	.00	.00	.00
	R^2	9%	12%	14%

Note. * Correlation is significant at the 0.05 level (2-tailed).
** Correlation is significant at the 0.01 level (2-tailed).

As shown on Table 19, the correlation between accommodating student needs and providing feedback and clarification to students was statistically significant ($r_{CCS\text{-}ASNS}$=.46, $R^2_{CCS\_ASNS}$=21.16%), indicating that if teachers feel that they can provide feedback to students they are able to accommodate individual student needs. The researcher conducted a correlational analysis for the items within the ASNS. The results, presented on Table 38 show that four of the five components had a weak relationship. In general, the data collected in this survey shows that teachers who feel they can motivate students also feel they can sometimes increase retention, redirect students and respond to student needs. They only two components which did not have a statistically significant relationship were the respondents' ability to get students interested in mathematics and the respondents' ability to increase student retention. The strongest correlations were between teachers' perceived ability to respond to student needs and their perceived ability to redirect students (r_{E18_E19}=.37, $R^2_{E18\_E19}$=14%). One item which did not load onto either the Clarification and Communication Scale nor the Accommodating Student Needs Scale was E2, "*Explaining manipulatives*". The factor loading for this item was .29 for component 1, Clarification and Communication, and .24 for component 2, Accommodating Student Needs (Appendix G).

The table below shows the frequency of responses for this item (E2). The data shows that 17.2% of respondents lacked confidence in their ability to use manipulatives to explain mathematical concepts.

Table 39 *Frequency of Responses "Respondents' ability to use manipulatives to explain mathematics concepts"*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Strongly Disagree	2	2.2	2.2	2.2
	Disagree	5	5.4	5.4	7.5
	Neutral	9	9.7	9.7	17.2
	Agree	27	29.0	29.0	46.2
	Strongly Agree	50	53.8	53.8	100.0
	Total	93	100.0	100.0	

Table 40 *Correlation Analysis: Using*

Manipulatives and Components of CCS

		Explaining Manipulatives
Explaining Manipulatives	*r*	1
	p	
Ability to Answer Student Questions	*r*	.21*
	p	.041
	R^2	4%
Welcome principal observation	*r*	.23*
	p	.03
	R^2	5%
Teacher's ability to explain concepts	*r*	.3**
	p	.004
	R^2	9%

Note. * Correlation is significant at the 0.05 level (2-tailed).
** Correlation is significant at the 0.01 level (2-tailed).

The correlation of item E2 with these two scales although statistically significant, was only weakly correlated to either component. (r_{E2_CCS}=.3, $R^2_{E2\_CCS}$= 9%; r_{E2_ASNS}=.29, $R^2_{E2\text{-}ASNS}$=8.4%). Therefore, it can be inferred that the respondents' ability to use manipulatives in the classroom was not a strong influence on their self- efficacy beliefs. A correlational analysis shows that using manipulatives to explain mathematical concepts had a significant statistical relationship with three of the five components of the Communications and Clarification Scale. The correlation between using manipulatives to explain mathematics and ability to answer student questions, ability to explain mathematics concepts and welcoming principal observations had a weak positive relationship. Therefore, emphasis on using manipulatives does not necessarily correlate with the respondents' perceived ability to provide clarification and feedback to their students.

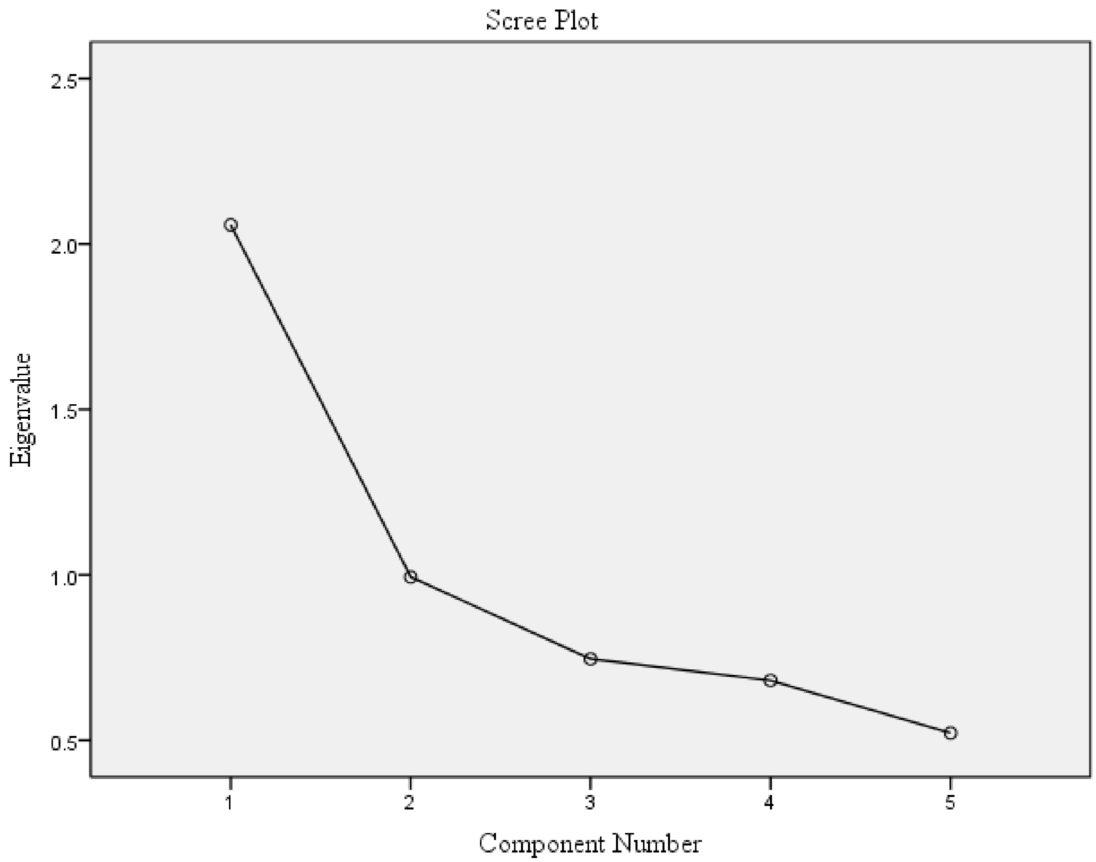
Research Question 2

Research question 2 asked *"What instructional strategies characterize those of elementary mathematics teachers?"*. The descriptive statistics for the variables created to answer this question, MIP_Adj and MIP_AdjM show a wide range in the mathematical instructional practices of respondents (Table 10). The mean score assigned to the respondents was reported by the variable, MIP_AdjM, which was created by taking the total score of the respondents and dividing by the number of items on the mathematics instructional practices scale (Table 8). Table 10 shows that the mean score for this variable was 2.69, reflecting more traditional, teacher- centered practices. As discussed in Chapter 4, the researcher also conducted a principle components analysis, thereby creating three subscales for mathematics instructional practices. These three subscales reflect the following: the use of creativity and higher order thinking skills, the use of discussion and the use of collaboration in the classroom (Table 11). The data presented on Table 12 shows the descriptive statistics for these components. The mean score among the respondents for the use of creativity and higher order thinking strategies was 2.54 which reflects the average response between "some lessons and "most lessons". The overall mean score for the use of discussion was 2.94, which reflects responses close to "most lessons". The average score among all respondents for the use of collaborative learning strategies was 2.41, which fell between "some lessons" and "most lessons, but was closer to "some lessons".

The data presented on Table 12 also shows that the total score for the use of creativity and gather order thinking skills (*M*=12.72) had a high variance of 6.05 points. The researcher was interested in further exploring this variance, since the eigenvalue of this component was 2.66 (Table 11) and accounted for 24% of the variance within the mathematics instructional practices of the respondents.

The scree plot shown in Figure 11 shows that two factors seem to have the most influence within the creativity and higher order thinking scale. A principle components analysis of this scale shows that three items have the most influence and have an eigenvalue of 2.06.

Figure 11 *Scree plot: Components within CHOTS*



These three items which make up 41.16% of the variance in the use of creativity and higher order thinking skills. The items are P10, P9, and P8 (Table 42).

Table 41 *Principle Components within CHOTS*

Item	Description	Factor Loading
P10	Use computers to complete exercises or solve problems	.79
P9	Work on problems for which there are no immediately obvious methods of solution	.68
P8	Represent and analyze relationships using tables charts or graphs	.76

The frequency of responses for item P 8, "*Students represent and analyze relationships using charts or graphs*" is presented on Table 42. The mean for this item was 2.6, the standard deviation was .63 and the variance was .39. The data shows that 4.3% of respondents never use this strategy, and 34.4 % use this strategy for "some lessons". 60.2% of the respondents use this strategy for most or every lesson. This is an area in which increased support by educational leadership is needed. This could come in the form of professional development or preparation for pre-service teachers. Interestingly, this is item had a significant statistical relationship with explaining mathematical concepts (r_{E2P8} =.21, R^2_{E2P8} =4%). The effect size for this relationship is small, only 4%, indicating that for the respondents of this survey, they do not find manipulatives the best way to explain mathematical concepts.

The frequency of responses for item P9, *"Students work on problems for which there are no immediate or obvious solutions"* (M = 2.2, SD= .83, variance= .69) is shown on Table 43. The data presented shows that 22.6% of respondents never or almost never use this strategy in their mathematics instruction, and 36.6% of respondents use it for some lessons. The strategy is at the heart of what experts call "inquiry-based learning". This strategy encourages students to use creativity, take risks, and look more deeply at mathematical concepts.

Table 42 *Frequency of Responses "Student represent and analyze relationships using tables charts or graphs"*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Never or Almost Never	4	4.3	4.3	4.3
	Some Lessons	32	34.4	34.4	38.7
	2.6	1	1.1	1.1	39.8
	Most Lessons	53	57.0	57.0	96.8
	Every Lesson	3	3.2	3.2	100.0
	Total	93	100.0	100.0	

Table 43 Frequency of Responses: *"Students work on problems for which there are no immediately obvious methods of solution"*

	Frequency	Percent	Valid Percent	Cumulative Percent
Never or Almost Never	21	22.6	22.6	22.6
Some Lessons	34	36.6	36.6	59.1
Most Lessons	35	37.6	37.6	96.8
Every Lesson	3	3.2	3.2	100.0
Total	93	100.0	100.0	

Table 44 *Frequency of Responses: "Students use computers to complete exercises or solve problems"*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Never or Almost Never	29	31.2	31.2	31.2
	Some Lessons	32	34.4	34.4	65.6
	Most Lessons	26	28.0	28.0	93.5
	Every Lesson	6	6.5	6.5	100.0
	Total	93	100.0	100.0	

The frequency of responses for item P10, *"Students use computers to complete exercises or solve problems"* is presented on Table 44. The data shows that 31.2% of respondents never or almost never have students use computers to solve mathematical problems, and another 34.4% have students use them for some lessons. Only 6.5% of respondents have students use computers to solve problems in every lesson. This is an area in which teachers obviously need support.

To measure the use of discussion used by respondents, the researcher created the variable, DCS (*M*=11.75, *SD*= 2.03, variance = 4.11). The variable DCS_M was created to better understand overall use of discussion. The total score in DCS was divided by the number of items. Table 12 show the descriptive statistics for this variable: *M*= 2.94, *SD*= .51, variance = .26. This reflects that teachers use discussion in some to most lessons. The following tables show that there is a mix of strategies most likely going on within the lessons that teachers conduct.

Table 45 F*requency of Responses: "Students work individually without assistance of the teacher"*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Never or Almost Never	3	3.2	3.2	3.2
	Some Lessons	29	31.2	31.2	34.4
	Most Lessons	46	49.5	49.5	83.9
	Every Lesson	15	16.1	16.1	100.0
	Total	93	100.0	100.0	

Table 46 *Frequency of Responses: "Students work individually with assistance from the teacher"*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Some Lessons	21	22.6	22.6	22.6
	Most Lessons	48	51.6	51.6	74.2
	Every Lesson	24	25.8	25.8	100.0
	Total	93	100.0	100.0	

As shown in Table 45, 495% of respondents have students work independently without the teacher's assistance for "most lessons", and 16.1% of respondents use this strategy for "every lesson". Table 46 shows that 77.4% of respondents have students work individually with assistance of the teacher for most to every lesson. This shows that although some reliance on the traditional method of teaching is used as reflected by item P1 in Table 46, the use of discussion with a teacher allows for the Zone of Proximal Development to encourage and clarify meaning for students.

Table 47 *Frequency of Responses: "*Work *together as a class with the teacher teaching the whole class"*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Never or Almost Never	4	4.3	4.3	4.3
	Some Lessons	17	18.3	18.3	22.6
	Most Lessons	41	44.1	44.1	66.7
	Every Lesson	31	33.3	33.3	100.0
	Total	93	100.0	100.0	

The data presented on Tables 47 and 48 indicate that the 77.4% of respondents use whole class instruction for most or every lesson, and 70.9% of respondents use whole class instruction with students responding to one another for most or every lesson. Interestingly, the students working together as a class with students responding to one another has a statistically significant relationship ($p<.05$) with teacher's perceived ability to get students interested in mathematics ($r_{P4\text{-}E15}=.26$, $R^2_{P4E15}=7\%$). This shows a weak relationship which can be used to enhance a classroom culture of inquiry within a classroom. Having students take risks to explore mathematics and respond to one another's ideas can help get the students interested in mathematics (Woods, 2017).

Table 48 *Frequency of Responses: "Work together as a class with the students responding to one another"*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Never or Almost Never	1	1.1	1.1	1.1
	Some Lessons	25	26.9	26.9	28.0
	2.9	1	1.1	1.1	29.0
	Most Lessons	51	54.8	54.8	83.9
	Every Lesson	15	16.1	16.1	100.0
	Total	93	100.0	100.0	

The use of collaboration used in the classroom was measured by the items P5, P6, P8, P9 and P10. The researcher was interested in exploring the items P5 "Students work in small groups without assisting each other" and P6 "Students work in small groups assisting each other.

The data presented on Tables 50 and 51 indicate that this is also a strategy used in conjunction with other strategies. For instance, Table 49 shows that 67.7% of respondents have students work in small groups that respond to one another.

There is an obvious overlap of strategies within the mathematics classroom. Additionally, these two strategies have as moderate, statistically significant relationship with each other

($r_{P5\text{-}P6}$=.4, $R^2_{P5\text{-}P6}$=16%), but do not correlate with other components of the Communication and Clarification Scale, nor the Accommodating Student Needs Scale.

Table 49 *Frequency of Responses: "Work in pairs or small groups without assistance from each other"*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Never or Almost Never	19	20.4	20.4	20.4
	Some Lessons	31	33.3	33.3	53.8
	2.3	2	2.2	2.2	55.9
	Most Lessons	35	37.6	37.6	93.5
	Every Lesson	6	6.5	6.5	100.0
	Total	93	100.0	100.0	

Table 50 *Frequency of responses: "Work in pairs or small groups without assistance from each other"*

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Never or Almost Never	19	20.4	20.4	20.4
	Some Lessons	31	33.3	33.3	53.8
	2.3	2	2.2	2.2	55.9
	Most Lessons	35	37.6	37.6	93.5
	Every Lesson	6	6.5	6.5	100.0
	Total	93	100.0	100.0	

Research Question 3

Research Question three asked *"What are teachers' beliefs about student learning and mathematics instruction?"* The researcher created a mathematics beliefs inventory (Table 14) to assess the respondents' beliefs. The data presented on Table 16 shows a wide range of beliefs about how mathematics is best learned. To reiterate, a higher score on the inventory indicated that respondents believed that teachers should focus on the cognitive processes of students; that learning should be inquiry- oriented. A lower scored indicated that traditional beliefs about mathematics instruction: mathematics is a fixed body of knowledge and that students should focus on practice of computation and follow rules without deeper understanding of concepts.

The mean score of the mathematics inventory was 54.81. This score had a variance of 50.08 points. Figure 12 shows the dispersion of total scores for the respondents of the survey.

The mean score for respondents was reflected in the variable MBI_AdjM. The mean of this variable was 3.04. In general, the respondents fell in the middle between traditional beliefs, and inquiry based beliefs about student learning.

Figure 12 *Histogram: Total Scores for Mathematical Beliefs Inventory*

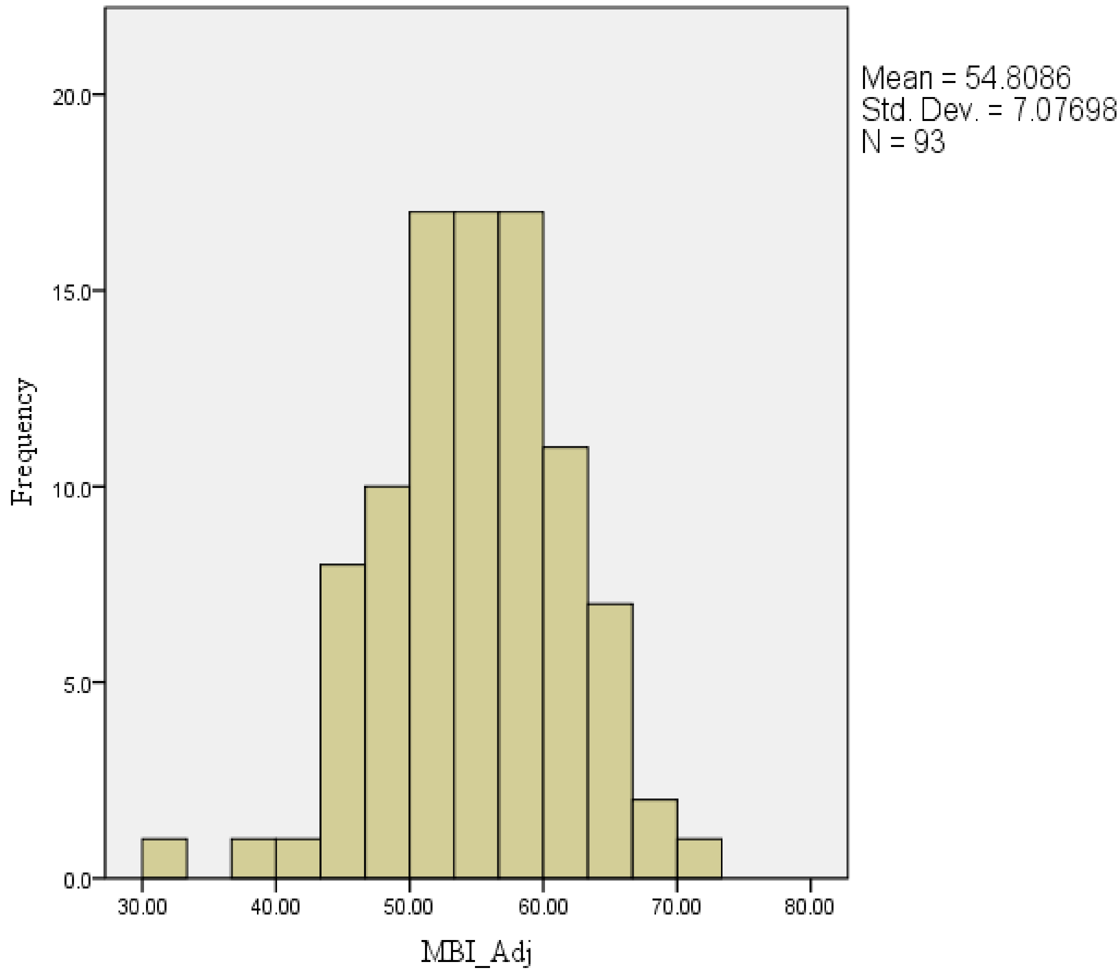
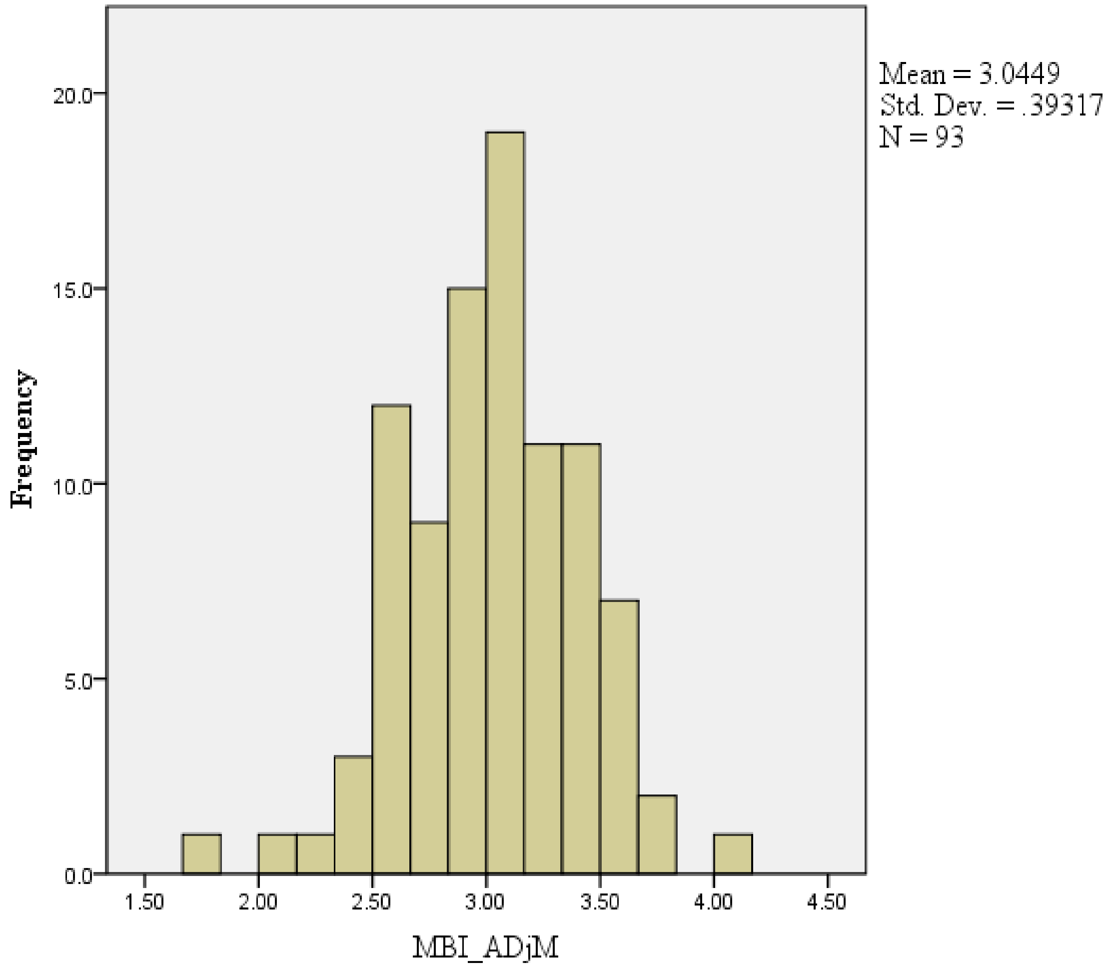
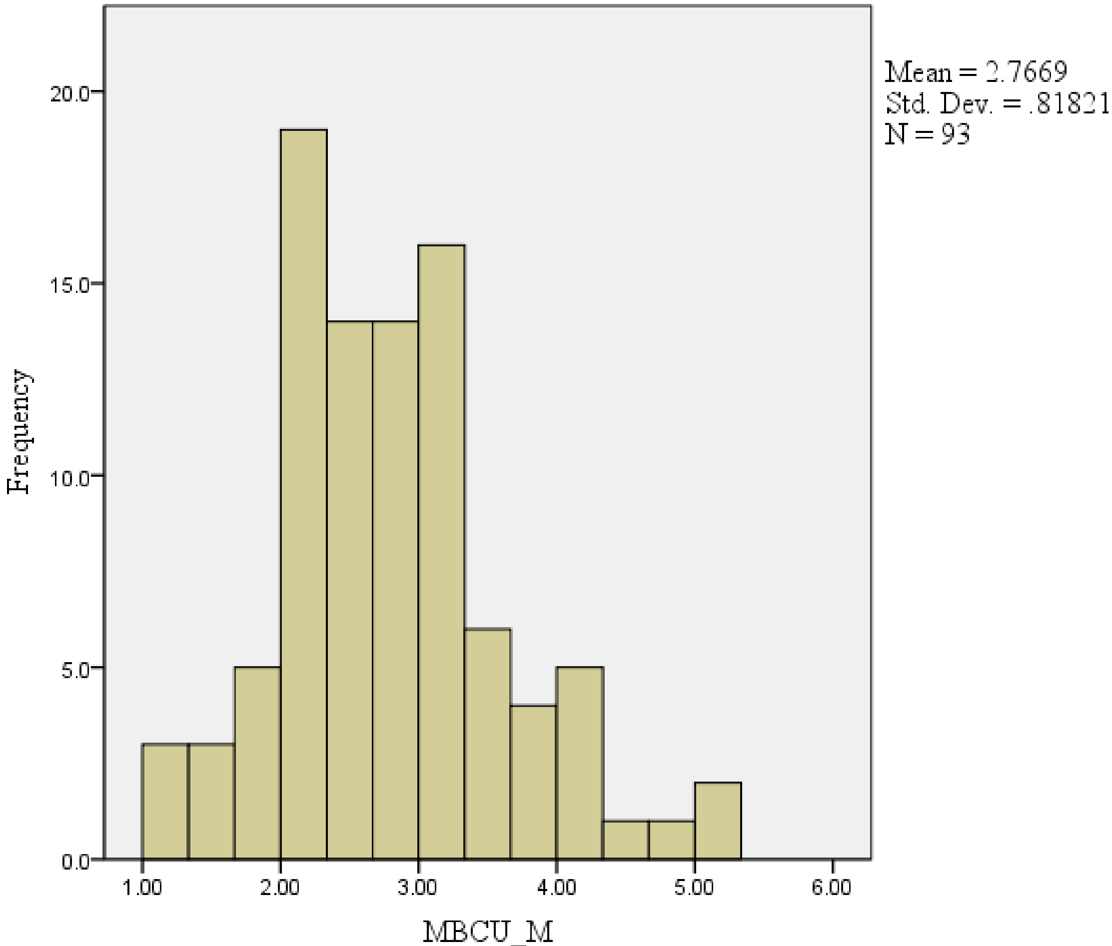


Figure 13 *Histogram: Mean Scores for Mathematical Beliefs Inventory*



The researcher conducted a principle components analysis for the mathematical beliefs inventory. Five items were identified as a subscale for the mathematics beliefs inventory. These items accounted for 18% of the variance within the mathematical beliefs inventory. The subscale was labeled Mathematical Beliefs of Conceptual Understanding. The data presented on Table 17 indicates that overall, teachers have traditional beliefs about conceptual understanding of mathematics; that is, students should practice mathematics before they understand underlying concepts.

Figure 14 *Histogram: Mathematical Beliefs of Conceptual Understanding (Mean Score)*



Research Question 4

Research question 4 asked "*what is the relationship between reflection in action, classroom discourse, and teacher's self- efficacy in mathematics instruction?"* To answer this question, researcher first conducted a correlational analysis between the two components of the mathematics self- efficacy scale (Table 19). The results indicate that there is a moderate, direct relationship between these two components. Earlier in this chapter, the researcher also conducted a correlational analysis among the components of the communication and clarification subscale (Table 32). To reiterate the findings presented on Table 32, the correlations were significant ($p < .01$). The correlation between a teacher's ability to answer student questions and welcoming student questions have a moderate, direct relationship. The strongest relationship, as shown in Table 32, is between the teacher's ability to explain concepts and welcoming student questions ($r_{E5-E13} = .5$, $R^2_{E5-E13} = 25\%$).

From this data, we can infer that if teachers feel confident in their understanding of mathematical concepts, they will also feel more confident in their ability to answer student questions, and accommodate individual student needs.

The researcher also conducted a correlational analysis for the items within the ASNS (Table 38). In general, the data collected in this survey shows that teachers who feel they can motivate students also feel they can sometimes increase retention, redirect students and respond to student needs. They only two components which did not have a statistically significant relationship were the respondents' ability to get students interested in mathematics and the respondents' ability to increase student retention. The strongest correlations were between teachers' perceived ability to respond to student needs and their perceived ability to redirect students ($r_{E18-E19}=.37$, $R^2_{E18-E19}=14\%$).

The need for teachers to feel more confident in their understanding of mathematical concepts was further explored through a correlational analysis regarding the use of manipulatives and teacher's perceived ability to provide feedback to students (Table 40). The data shows that the use of manipulative had a significant relationship with only two of the components of the clarification and communication scale. The use of manipulative had a weak direct relationship with respondents' perceived ability to explain concepts

($r_{E2\text{-}E13}$=.3, $R^2_{E2\text{-}E13}$=9%), and teacher's perceived ability to answer student questions ($r_{E2\text{-}E4}$=.21, $R^2_{E2\text{-}E4}$= 4%). There is no correlation between the teacher's perceived understanding of mathematical concepts and their ability to use manipulatives to explain concepts. Therefore, the researcher can infer that more support is needed to help teachers use manipulatives more productively in the classroom, or find other ways to help teachers explain underlying mathematical concepts to students. Additionally, the teacher's perceived ability to explain concepts (E6) and their use of the instructional practice in which students must explain the reasoning behind the ideas (P7) had a statistically significant relationship (p=.01, $r_{p7\text{-}E6}$=.27, $R^2_{p7\text{-}E6}$=7%). The data therefore suggests that if teachers feel confident in their ability to explain concepts, they will be slightly more likely to require students to explain the reasoning behind their solutions to problems.

The researcher conducted a correlational analysis among the three components of the instructional practices variable (Table 21). There was a positive, low relationship between the use of discussion in the classroom and the use of creativity and higher order thinking skills, but there was a strong direct relationship between the use of collaboration and creativity/higher order thinking skills. ($r_{CHOTS\text{-}DCS}$ = .28, $R^2_{CHOTS\text{-}DCS}$ = 8%; $r_{CHOTS\text{-}CIP}$=.76, $R^2_{CHOTS\text{-}CIP}$= 58%). The data suggests that teachers who use open- ended questions and inquiry learning will also use collaboration within the classroom.

The researcher conducted further correlational analysis regarding mathematics instructional practices. The researcher conducted a correlational analysis between the use of the two strategies P1 and P2 (Tables 45 and 46) with the components of the Clarification and Communication Scale (Table 6). While the two practices have a weak statistically significant relationship ($p<.01$, $r_{P1\text{-}P2}$=.35, $R^2_{P1\text{-}P2}$=12%), it is important to note that these strategies do not have a significant relationship with any of the five components of the CCS scale.

The researcher also conducted a correlational analysis between the use of the two strategies P1 and P2 (Table 45 and 46) with the components of the Accommodating Student Needs Scale (Table 6). The correlational analysis indicated that there was a statistically significant relationship ($p<.05$) between P1 "*Students work individually without assistance from the teacher*" and E16, "*Respondents' ability to motivate difficult students*" ($r_{P1-E16}=.22$,

$R^2_{P1-E16}=5\%$). If a teacher feels confident in their ability to motivate difficult students they may be more likely allow students to work independently without assistance.

The researcher then examined the correlation between the two other items which were part of the discussion subscale: items P3 and P4 (Table 8). These items reflect whole class instruction, with and without students responding to one another. These items had a statistically significant relationship ($p=.38$, $r_{p3-p4}=14\%$). This data indicates that if a teacher uses whole class instruction there is a weak direct relationship that the class will discuss the topics together as a class. The researcher also conducted a correlational analysis of P3 and P4 with the components of the CCS (Table 6). P3, *"Students work together as a class with the teacher teaching the whole class"* did not correlate with any of the components of the communication and clarification scale. However, P4, *"Students work together with a class responding to one another"* had a weak statistically significant relationship with E11 *"Teacher welcomes principal observation of mathematics lessons"* ($p=.038$, $r_{P4-E11}=.22$, $R^2=5\%$). From this data, we can infer that if a teacher uses whole class instruction they will be only slightly more likely to welcome a principal's observation of their mathematics lesson.

A correlational analysis was conducted between P3 and P4 with the components of the accommodating individual needs scale (Table 6). P3 had a statistically significant relationship (*p*=.031) with only one component, E17. Therefore, if a teacher feels confident in their ability to increase student retention, they may be slightly more likely to use whole class instruction without students responding to one another ($r_{p3\text{-}E17}$=.22, $R^2_{p3\text{-}E17}$=5%). P4 had a statistically significant relationship with two components (*p*<.05). If a teacher feels confident in their ability to get students interested in mathematics (E15), they will be slightly more likely to use whole class instruction with students responding to one another ($r_{p4\text{-}E16}$=.22, $R^2_{p4\text{-}E16}$=5%). As noted earlier, a teacher's perceived ability to get students interested in mathematics also meant that they would be more likely to welcome a principal's observation, although the relationship was weak (*p*=.02, $r_{E16\text{-}E11}$=.24, $R^2_{E16\text{-}E11}$=6%). Teachers who feel able to motivate students, in a class where students are interacting in a vibrant way would certainly be more likely to welcome the principal's observation, but as noted on Table 32, there are other factors which more strongly influence a teacher's confidence in being observed by their principal.

In order to better understand the factors affecting the use of collaboration in the mathematics classroom, the researcher conducted a correlational analysis between items P5, *"Students work in small groups without assistance from each other",* and P6, *"Students work in small groups with assistance from each other",* with the components of both the CCS and ASNS (Table 6). Although these two components have a moderate direct relationship with each other (*p* <.01, $r_{p5\text{-}p6}$ =.4, $R_{p5\text{-}p6}$=16%), they do not have a statistically significant relationship with and of the components of the CCS or ASNS.

The data presented on Table 21 indicated that there was a statistically significant relationship between mathematical beliefs and mathematical instructional practices ($r_{MBI-MIP}$=.25, $R^2_{MBI-MIP}$=6%). As discussed in Chapter 4, this indicated a weak direct relationship. The data presented on Table 22 showed a weak positive relationship with mathematical beliefs and the use of creativity and higher order thinking skills ($r_{MBI-CHOTS}$=.26, $R^2_{MBI-CHOTS}$=7%) The researcher conducted further correlational analysis for this discussion to find out which components of mathematical belief inventory (Table 13) had a significant relationship with the subscale for creativity and higher order thinking skills (Table 11). It is interesting to note that the subscale for mathematical beliefs about conceptual understanding (MBCU) did not have a significant relationship with the use of creativity and higher order thinking skills.

The correlational analysis showed that MB1 *"Teachers should encourage students to find their own solutions to math problems even if they are inefficient"* had a weak direct relationship with two of the components of the creativity and higher order thinking scale: P7, *"Students explain the reasoning behind an idea"* (p=.018, r_{MB1-P7}=.25, R^2_{MB1-P7}=6%), and P11, *"Students write equations to represent relationships"* (p<.01, $r_{MB1-P11}$=.33, $R^2_{Mb1-P11}$=11%). The data indicates that if a teacher believes that students should construct meaning from their experiences, they may be slightly more likely to encourage students to write equations and explain their reasoning for their solutions. The researcher also must acknowledge that this weak relationship indicates that there must be other factors that will influence a teacher's choice to use these instructional strategies.

P11 also had a direct, weak relationship with MB4, *"Students should master computational procedures before they are expected to understand how those procedures* *work"* (*$p<.05$, $r_{MB4-P11}$*=.21, $R^2_{MB4-P11}$=4%), MB8, *"Most students can figure out ways to solve many mathematical problems"* ($p<.05$, $r_{MB8-P11}$=.21, $R^2_{MB8-P11}$=4%) and MB14, "*Mathematics should be presented to students in a way that they can discover relationships for themselves"* (*$p<.05$, $r_{MB14-P11}$*=.22, $R^2_{Mb14-P11}$=5%). This is interesting data. It can be inferred that the use of P11, writing equations is related to the belief that computation should proceed understanding and the belief that students should discover relationships for themselves, which seem to contradict each other.

The instructional practice P8 *"Students represent and analyze relationships using tables charts or graphs"* had a statistically significant relationship with three of the components of the Mathematical Beliefs Inventory. The data indicates that this component of the Creativity and Higher Order Thinking Scale had a weak, direct relationship with MB5, *"Students need explicit instruction on how to solve word problems"* (*$p<.01$, r_{MB5-P8}*=.27, R^2_{MB5-P8}= 7%), MB6, *"Teachers should allow students to argue out their own ways to solve word problems"* ($p<.05$. r_{MB6-P8}=.2, R^2_{MB6-P8}=4%), and MB 14, "*Mathematics should be presented to children in such a way that they can discover relationships for themselves"* ($p<.05$, $r_{MB14-P8}$=.25, $R^2_{MB14-P8}$=6%). This practice is therefore associated with beliefs which seem contraindicative. Therefore, it is obvious that further research, perhaps in the form of interviews and observation could further explain how the students use tables, charts and graphs in the classroom.

The instructional practice, P9, *"Students work on problems for which there are no immediately obvious methods of solution"* had a statistically significant relationship with three components of the Mathematical Beliefs Inventory, MB7, "*The goals* of *instruction in mathematics are best achieved when students find their own methods for solving* *problems*" (p<.05, r_{MB7-P9}=.25, R^2_{MB7-P9}=6%) and MB8, "*Most students can figure out ways to solve many mathematical problems"* (p<..05, r_{MB8-P9}=.23, R^2_{MB8-P9}=5%) MB18, *"Teachers should allow students who are having difficulty solving a word problem to continue to try to find a solution"* (p<.05, $r_{MB18-P9}$=.21, R^2=4%). These correlations indicate weak direct relationships. Again, the use of such inquiry- based practices must be influenced by more than just the beliefs of the instructor.

Lastly, P10, *"Students use computers to complete exercises or solve problems"* did not have a statistically significant relationship with any of the components of the Mathematical Beliefs Inventory.

In Chapter 4, the researcher conducted a correlational analysis between the two subscales associated with mathematics teaching self- efficacy beliefs. The data showed a significant statistical relationship between mathematical beliefs and the clarification and communication scale (Table 23). A closer look at the components of the two scales show that not all the components have statistically significant relationships. E4 *"Ability to answer student questions"* does not have a statistically significant relationship with any of the components of the Mathematical Beliefs Inventory, but E5, *"Welcome student questions"* has a weak direct relationship with MB12, *"Students must be good listeners"* (p<.05, $r_{E5-MB12}$=.22, $R^2_{E5-MB12}$=5%). Table 32 showed that the teacher's perceived ability to explain concepts influenced the teacher's welcoming student questions, and now we have learned another factor which influences this part of teacher's self- efficacy beliefs, whether students have been good listeners in class.

Data presented on Table 52 shows that E6, "*Teacher's understanding of mathematical concepts"* had a statistically significant relationship with six of the components of the

Mathematical Beliefs Inventory. Although these are all weak relationships, the most notable was with a teacher's belief that students can figure out ways to solve problems. In general, if a teacher feels confident in their understanding of mathematical concepts, they may also hold a range of beliefs, from a traditional focus on computational accuracy, to that of an inquiry- based approach.

Table 51 *Correlation: Teacher's perceived understanding of mathematical concepts and mathematical beliefs inventory.*

		Understanding of Mathematical Concepts
Encourage students to find their own solutions to math problems even if they are inefficient	*r*	.26*
	p	.014
	R^2(%)	6%
Students should master computational procedures before they are expected to understand how those procedures work.	*r*	.22*
	p	.033
	R^2(%)	5%
The goals of instruction in mathematics are best achieved when students find their own methods for solving problems.	*r*	.22*
	p	.031
	R^2(%)	5%
Most students can figure out ways to solve many mathematical problems	*r*	.32**
	p	.002
	R^2(%)	10%
Time should be spent practicing computational procedures before students are expected to understand the procedures.	*r*	.21*
	p	.045
	R^2(%)	4%
Students should not solve simple word problems until they have mastered some number facts.	*r*	.23*
	p	.024
	R^2(%)	5%

*Note**. Correlation is significant at the 0.05 level (2-tailed).
** Correlation is significant at the 0.01 level (2-tailed).

The self- efficacy belief E13 *"Teacher's ability to explain concepts"* had a statistically significant relationship with only one component of the mathematical beliefs inventory, MB18, *"Teachers should allow students who are having difficulty solving a word problem to continue to try to find a solution*". This is a weak relationship (p<.01, $r_{E13\text{-}MB18}$=.28, $R^2_{E13\text{-}MB18}$-8%), but in general, if a teacher feels able to explain concepts they may also believe that students should persevere in efforts to find solutions to difficult problems. This may also indicate a teacher will be slightly more confident to explain word problems with their students, focusing on their cognitive processes and creative solutions.

Lastly, the correlational analysis did not show a statistically significant relationship between a teacher's welcoming of the principal's observation of mathematics lessons (E11) and any of the components of the mathematical beliefs inventory scale (Table 13).

Summary and Recommendations

The results of this study have shown that instituting new instructional strategies themselves will not improve teacher self- efficacy. To improve self- efficacy and establish a practice of student- centered learning, underlying beliefs of teachers must change, and their understanding of mathematical concepts must be strengthened and supported.

The data presented in this study shows that the key factors affecting teacher self-efficacy were the teachers' perceived understanding of mathematical concepts and their ability to answers student questions. There was also a correlation between a teacher's ability to explain concepts and welcoming student questions (Table 32). A strong understanding of concepts is necessary to provide reflective feedback to students. The data presented in this study show that although the respondents of this survey may welcome student questions, they do not always feel confident in their ability to answer the questions posed. The three components that account for over half of the variance in respondents' confidence in communicating with students were their ability to understanding concepts, welcoming and answering questions (Table 31).

These factors are also moderately related to the respondents' ability to accommodate individual student needs (Table 19). In general, if a teacher feels confident that they can provide feedback, they also feel that they can accommodate student needs. For the data collected, teachers feel least confident in their ability to motivate difficult students, followed by their ability to respond to student needs (Tables 34 and 37). These two items are also weakly correlated. In general, for the data collected, if a teacher can redirect difficult students (E18) they also feel that they can respond to student needs (Table 38).

This data confirms and explains previous studies found in Chapter 2 of this study. Nurlu's findings (2015) that teachers with high mathematics self- efficacy beliefs are more open to innovative ideas and methods which will accommodate student needs. The underlying factor is teachers' confidence in understanding and communicating mathematical concepts. Also, as originally found by Hughes (2016), teachers with low perceived self- efficacy teach in traditional ways. This study found that the underlying reason was a lack of confidence in answering student questions related to their perceived understanding of mathematical concepts.

Previous research has shown that the use of manipulatives can help a child construct meaning, understand concept, provide the basis for social learning as they explore and talk about their actions (Weber, 2005). The use of manipulatives has not been shown to be an influential means of communicating mathematical concepts for the respondents of this survey (Tables 39 and 41). As Weber (2005) observed, "manipulatives themselves do not teach, and therefore skillful teachers need to recognize when and where they can be used" (page 34). Therefore, this is an area which needs to be addressed by educational leaders.

As stated earlier in this paper, computers are not often used in the mathematics classroom. In the 2016 report *Future Ready Learning*, the National Educational Technology Learning Plan (NETP) written by the US Dept. of Ed, Office of Educational Technology, separates learners who use technology in creative ways and those who use it for passive consumption (p. 5). The findings of this study demonstrate that this digital divide can exist within schools, among teachers of different disciplines. This is also an area that needs to be addressed by educational leaders.

Also discussed earlier in this paper, Boaler and Dweck (2016) observed that current teacher preparation programs fail to adequately prepare teachers for mathematics instruction. The data from this study confirms and explains that to improve teacher's understanding of concepts and improve confidence in answering student questions, it is important that mathematics be given a more prominent role in elementary teacher preparation programs. Attention must be given to the type of instruction provided to both preservice teachers, as well as those already in the classroom. Pedagogical knowledge is not enough, conceptual knowledge must be supported.

The needs of adult learners are different from those of children, and although this is not within the scope of this study, there are important points that will be noted here.

It has been established that adult learners are self-directed and more subject centered

(Knowles, Holton & Swanson, 2005). Additionally, they need to know the reason for learning. As adults, their reservoir of experience can not only facilitate learning, but also hinder learning. Adult learners with negative beliefs about their ability teach mathematics, or those who dislike mathematics may have had experiences that function as a barrier to learning. In their book, *Adult Learning: Linking Theory and Practice,* Merriman and Bierma stated that facilitators set a "climate for learning that physically and psychologically respects adult learners" (2013, p. 47). Due to life experiences, adult learners may become "dogmatic and close-minded about learning something new" (p. 50).

Therefore, teachers who are used to using traditional methods of teaching mathematics may see no need to learn something new, or may not feel comfortable with new methods of teaching. Merriman & Berima suggested that a "facilitator can begin with an adult students' experiences and then assist the learner to connect those experiences with new concepts, theories and experiences" (2013, p. 51). They also suggested using discussion, role play, simulation, field experiences, problem-based learning, case studies, and projects to engage learners, and draw on their life experiences as resources for learning. Guskey wrote that in order "to lead changes in practice and improved results with students… (professional development) must be accompanied by structured opportunities for practice and feedback, collaborative planning and ongoing assistance" (2000, p. 209). Significant changes in teachers' attitudes and beliefs occur after they gain evidence of improvements in student learning (Guskey, 2000, p. 139).

The principal should serve as a facilitator in the process of professional development, rather than an authority figure that makes decisions. It is notable that the data from this study showed that there was a statistically significant relationship between the respondents' perceived understanding of mathematical concepts and their confidence in being observed by their principal (Table 29). Yet, in their book, *"The Effective Principal",* Nelson and Sassi (2014) observed that principals do not have expertise in all subjects. Their mathematical knowledge and assumptions about how math should be taught influences what they observe, and suggestions they make for intervention (Nelson & Sassi, 2014; Spillane, Halverson & Diamond, 2004).

Mathematical knowledge of most administrators is mostly procedural; based on the traditional learning model based on "memorizing facts and procedures and reproducing them when required" (Nelson & Sassi, 2014, p. 13). Therefore, it is important that administrators develop deeper understanding of the nature of mathematical knowledge and learning as discussed in this study, so that they can "go beyond the surface features of instruction and to discuss with teachers what needs to happen for real learning to occur" (Nelson & Sassi, 2014, p. 31). As Guskey (2000) observed, everyone who affects student learning must be learning all the time. This includes not only teachers and principals, but also school administrators and district leaders.

In his webinar for edWeb.net, David Woods provided suggestions for teachers to develop a strong math culture within their classrooms and schools. Changing the culture of the classroom and school entails changing the way teachers interact within students in the classroom, the discussions, questions, and projects that students participate in. Guskey (2000) observed that "changing school culture… requires the development of new values, beliefs and norms… it often involves building commitment to continuous learning and problem solving through collaboration" (p. 151).

Again, this brings the discussion back to educational leaders such as principals and district administrators. Their beliefs about the nature of mathematics, how it is learned and should be taught, not only affect how they "become engaged with issues of learning and teaching in their schools" (Nelson & Sassi, 2014, p. 123), but also how they develop a vision for their school as well as how they as educational leaders engage the school community to move toward that vision.

A change in the mathematical culture of a school will take the participation of not only teachers in their classrooms, but also principals, administrators and the parents. Smith (2011) suggests that the process of change should begin with an assessment of the school's current situation using students' current mathematic performance. If the school determines that the quality of mathematical performance is far from efficacious, then the school will need to consider discontinuing its model and designing a new model of schooling.

Educational leaders need to encourage "unfreezing" entrenched ideas through not only dialogue about innovative ideas, but also study groups in which participants present objective evidence for the issues they raise (Bernato, 2017; Deal & Peterson, 1999; Nelson & Sassi). By doing so, the group will see the need to engage in new habits of thought, recognize new opportunities, "use the creativity of the whole to create the preferable future" (Bernato, 2017, p.110).

For Further Research

Although the data from the survey was exceptionally rich, the limitations of this survey encompassed several items. It did not collect data regarding the years of experience of the respondents. This is certainly an area which needs further study. Secondly, the survey should also be given to a variety of schools- parochial, private, charter and public, in various parts of the country. The researcher could then compare the results based on the type of school, the socioeconomic background of the district and the experience of teachers.

Although the survey collected data regarding the grades taught by the respondents and the gender of respondents, this was not within the scope of the current study. This is also an area which deserves further research. Also, as mentioned in Chapter 1, the study did not focus on students with dyscalculia. The researcher recommends that research is needed which will support classroom teachers best attend to the needs of these students.

Furthermore, the survey did collect data regarding outcome expectancies of the respondents (Table 2). This data was not used for the current study, but is it is an area of interest for the researcher. Future research should explore the relationship between outcome expectancies, mathematics instructional practices and mathematical beliefs. This could also be done with a variety of schools to see if the outcome expectancies of teachers are different based on the population of the student body. The survey should be given on a test- retest basis before and after the recommended professional development was conducted by the educational leadership of a school or school district.

The original conceptual framework, found in Chapter 2, included two aspects, reflection in action, and classroom environment. The data from the survey reflected teachers' perception of their classroom environment and their reflection on student work. If the study had also included interviews and observations of a random sample of the respondents, the researcher could have gleaned insight into the self- efficacy beliefs, as well as captured nuances of social interactions in the classroom and how they relate to the mathematics instructional beliefs of the teachers. Interviews could also help explore element of cognitive dissonance and its effect on the respondents of this study.

Additionally, the role of principals and administrators must be further examined. As Nelson and Sassi (2014) observed, principals must be able to connect mathematical ideas to teachers' practice. The type of observations that are done must include the principal's ability to "attend to the particulars of teachers practice and help teachers cultivate a particular kind of attention to their students' mathematical thinking" (p.75). Principals need to be able to identify those who have that knowledge. Dr. Spillane of Northwestern University described a distributed perspective of leadership which focuses on not only leadership function, but of leadership practices stretched over two or more leaders (formal and informal), followers and artifacts (Spillane et al., 2004). Distributed leadership practices can be a resolution to the issues discussed above. As Bernato (2017) observed, "schools are too complex for one person to lead independently" (p. 39), and where individual leaders do not have the "requisite knowledge for the task at hand", (Nelson & Sassi, 2014, p. 76) they must be able to identify those who have that knowledge and how to use it.

To make improvement in student learning, educational leaders need quantitative data which is quite detailed (Bernato, 2017, p. 86) and "attuned to the particularities of each teacher's instructional practices" (Nelson & Sassi, 2014, p. 97). The data presented study can be a starting point for educational leaders.

Epilogue

Reflecting upon the data found in this survey, the researcher has developed the conceptual framework shown in Figure 15, which shows the relationship between instructional practices, mathematics self- efficacy beliefs, and mathematics instructional beliefs. As discussed earlier in this chapter, respondents' understanding and confidence in explaining mathematical concepts impacts all the components within the framework.

What does it all mean? From this study, it has become apparent to the researcher that although teachers acknowledge the need for a dialogue in the classroom, they do not feel prepared to delve deeply into the concepts that underlie mathematical computation. As stated in the recommendation section, teacher preparation needs to require more mathematics in teachers' education, as well as methods classes with opportunities for field work in settings that use instruction which is conceptually- based.

In conducting research for the literature review found in Chapter 2, the researcher found a notable example found in the study conducted by Sparrow & Frid (2009), who observed that to "break the cycle of tradition" and foster a classroom as a place rich in discourse about mathematical concepts and meaning, pre-service teachers needed to "mathematics content knowledge, pedagogical competence and mathematics professional confidence" at graduation (p. 37).

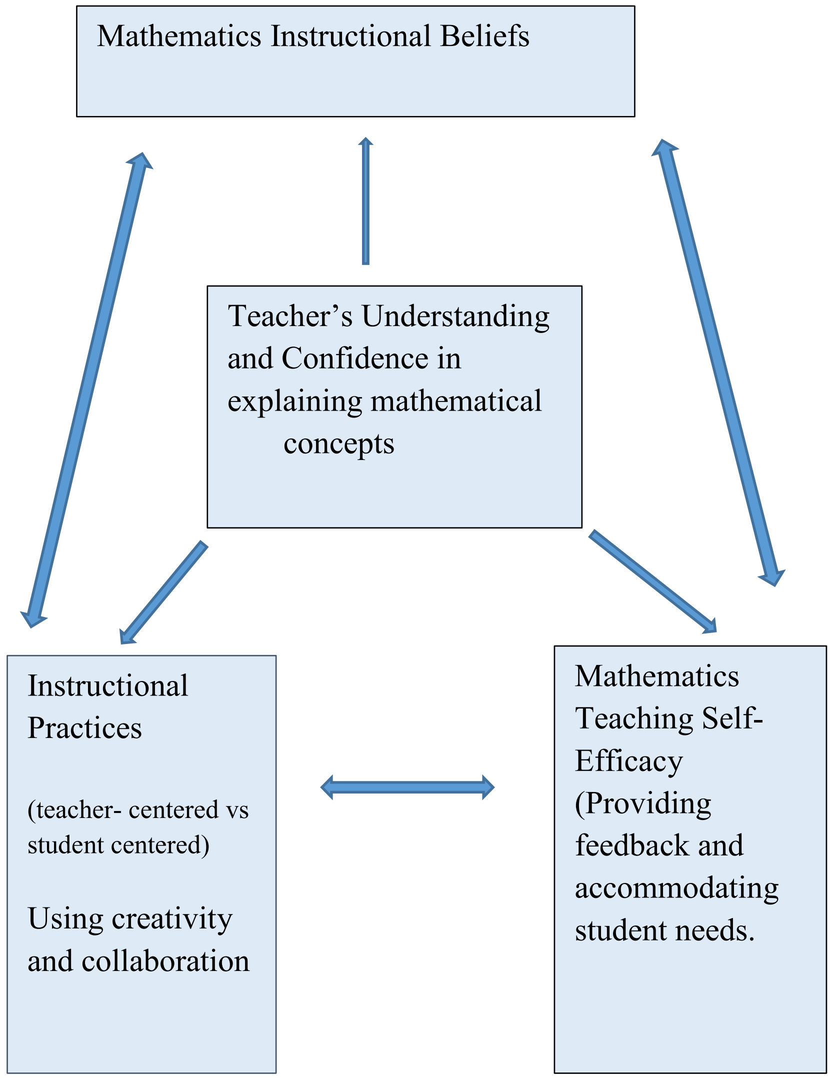


Figure 15 *Revised Conceptual Framework* .

In lieu of changing the teacher preparation programs across the country, a practical application of the findings of this study is that schools should put more emphasis on supporting elementary teachers in their mathematics instruction. The respondents in this study expressed a lack of confidence in their performance in front of their superiors. How can we therefore open the conversation among teachers and improve instruction and self-efficacy? In response to the findings and suggestions in this study, it seems a distributed leadership perspective could help support teachers and move teachers in the direction of more student-centered learning. Districts could require an extension for the upper elementary grade teachers (fourth and fifth) who teach mathematics. This may not be a practical solution for the immediate future.

Spillane, et al., (2004) describe a distributed leadership model in which multiple leaders have a multiplicative effect "because the interactions among two or more leaders in carrying a particular task may amount to more than the sum of those leaders' practice" (p. 16). Instructional support of teachers can be shared by "teacher- leaders", principals and math specialists. While literacy coaches are common, math coaches, or mathematics specialists are rarely found in elementary school settings. Math coaches can offer classroom teachers professional development and "push- in" instructional support. In this way, leadership can be practiced in an informal situation, discussion techniques, supporting teachers' ability to respond to student needs, explain concepts, answer questions and motivate students by providing ideas and resources which will get teachers interested in mathematics. Observation and practice of methods using charts, graphs, manipulatives and computers could help teachers feel more confident in their ability to explain concepts to students, thereby moving away from a teacher- based traditional model of instruction to a more student- centered mathematics classroom, filled with inquiry, creativity, thereby sparking more interest in mathematics. "Small changes," Senge observed, "can produce big results" (p. 63, 2006). This incremental change is what will hopefully start the change in mathematics culture within schools and hopefully society at large.

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Appendix A
Permission to Use Research Instrument

5/23/2017 Mail - christine.pacinello16@my.stjohns.edu

Re: Permission to use Research Instrument

James P Spillane <j-spillane@northwestern.edu>

Wed 5/3/2017 2:19 PM

To:Christine L. Pacinello <christine.pacinello16@my.stjohns.edu>;

Cc:Melissa Ortiz <melissa.ortiz@northwestern.edu>;

thanks Christine
You have my permission
best
j

On May 3, 2017, at 1:18 PM, Christine L. Pacinello <christine.pacinello16@my.stjohns.edu> wrote:

Dear Dr. Spillane:
Thank you for suggesting that I check out more recent instruments on the Distributed Leadership site. I found the Instructional Interactions Survey, which was used in the Auburn Park/Twin Rivers study (2015), detailed in the article "Conceptualizing relations between IGI and teachers' beliefs about mathematics instruction" (2015).
I am interested in possibly using this instrument for my dissertation. Therefore, I am requesting permission to use this instrument.
Thank you again!
Christine

From: James P Spillane <j-spillane@northwestern.edu>
Sent: Monday, May 1, 2017 8:18:01 PM
To: Christine L. Pacinello
Subject: Re: Permission to use Research Instrument

thanks Christine
best
j

On May 1, 2017, at 2:04 PM, Christine L. Pacinello <christine.pacinello16@my.stjohns.edu> wrote:

Thank you so much for getting back to me. I appreciate it. Also, I greatly admire your work.
Yours,
Christine

From: James P Spillane <j-spillane@northwestern.edu>
Sent: Sunday, April 30, 2017 9:30:30 PM
To: Christine L. Pacinello
Subject: Re: Permission to use Research Instrument

Hi Christin
I think we used the TIMSS survey in its entirety so you will need to reach out to TIMSS office at Michigan State University or Boston College;
if you check out distributed leadership.org
you can find our most recent instruments
j

On Apr 29, 2017, at 10:13 AM, Christine L. Pacinello <christine.pacinello16@my.stjohns.edu> wrote:

Dear Dr. Spillane,
I am currently completing my Ed D. through St. John's University in Queens, NY. My dissertation topic is "Factors affecting teacher self- efficacy in elementary mathematics instruction". I will be exploring the relationship between instructional practices and their teachers' self efficacy beliefs. I am considering using the TIMSS questionnaire you used in your journal article, "Reform and Teaching: Exploring Patterns and Practice in the Context of National and State Mathematics Reforms". Therefore, I am requesting permission to use your instrument.

Thank you for your time.

Yours,
Christine Pacinello

James P. Spillane
Spencer T. & Ann W. Olin Professor of Learning & Organizational Change
Professor of Human Development & Social Policy
Professor of Learning Sciences
Faculty Associate Institute for Policy Research
Northwestern University
www.sesp.northwestern.edu
www.distributedleadership.org
twitter: @jamespspillane

James P. Spillane
Spencer T. & Ann W. Olin Professor of Learning & Organizational Change
Professor of Human Development & Social Policy

https://outlook.office.com/owa/?realm=my.stjohns.edu 1/2

Appendix B
Request to Conduct Survey

June 1, 2017

Dr. Kathleen Walsh

Superintendent of Schools

Diocese of Rockville Centre

128 Cherry Lane,

Hicksville, NY 11801

Dear Dr. Walsh:

I am a Doctoral student at St. John's University, in Queens, NY. I am writing to request your help to conduct research in an area that I believe will help mathematics education. As a mathematics teacher, I often discuss current trends in mathematics curriculum and instruction with administrators, colleagues and parents. The present literature revealed that not much research has been done to identify factors related to mathematics instruction which would influence teacher efficacy beliefs, and it highlighted the need for more research.

I will examine the relationship between mathematics teachers' use of instructional strategies and their level of self- efficacy. I will measure this using a survey which will be available online.

I am contacting you to seek permission to survey elementary teachers in June. If you grant this permission, I will provide you with the copy of the invitation to participate in my research. This invitation will be emailed to principals throughout the diocese, and a link to the Google survey will be available until the end of the school year. Teachers will complete the survey electronically and confidentiality will be maintained. Participants' email address, IP address and individual responses will not be identified nor tracked as part of data collection. I will share the results of this study with the diocese.

If you would like to preview the survey, please follow the link:
https://goo.gl/forms/6rDo7wkSI8GsDilm2

Thank you for considering my request. If you grant permission, please email the approval to me at christine.pacinello16@my.stjohns.edu. If you have any questions or concerns, please call me at (631) 404- 7768, or (631) 724-0285. The results of this study will increase the understanding of teacher self-efficacy beliefs, inform educational leader, and promote student learning.

Respectfully,

Appendix C

Permission to Survey Teachers in the Diocese of Rockville Cent

Reply all | ∨ Delete Junk | ∨ •••

Permission to Survey Teachers in the Diocese of Rockville Centre

AB Anthony Biscione <abiscione@drvc.org> Reply all | ∨

Today, 9:00 AM

Christine L. Pacinello

Inbox

You replied on 6/5/2017 2:03 PM.

Dear Mrs. Pacinello,

This is to confirm that our Superintendent, Dr. Kathleen Walsh, has given her consent for you to conduct your survey within the Catholic schools of the Diocese of Rockville Centre.

With best regards,
Anthony Biscione

Mr. Anthony Biscione
Assistant Superintendent for Curriculum, Instruction and Assessment
Department of Education
128 Cherry Lane
Hicksville, New York 11801
Phone: (516) 280-2479 x813
Fax: (516) 280-2963
Email: abiscione@drvc.org

Appendix D
Invitation and Consent to Participate in a Research Study

Invitation and Consent to Participate in a Research Study

Dear Elementary Teachers,

You have been invited to take part in a research study to learn more about factors that affect self- efficacy in elementary mathematics instruction.

I will be conducting this study as part of my doctoral dissertation for St. John's University, Department of Administrative and Instructional Leadership.

I am asking you to take a brief online survey. The information you provide will assist in determining if instructional practices affect teacher self- efficacy. The results of this study will increase the understanding of teacher self-efficacy beliefs, inform educational leader, and promote student learning.

This survey will take approximately 10 minutes of your time. Information collected will be confidential. The survey will be completely anonymous. Names, email addresses, and IP addresses will not be collected.

Please follow the link below to take the survey.

https://goo.gl/forms/6rDo7wkSI8GsDilm2

If you have any questions or need assistance, please email me at christine.pacinello16@my.stjohns.edu or you may contact my faculty sponsor, Dr. Anthony Annunziato at annunzia@stjohns.edu.

Thank you. I truly appreciate your participation in this study!

Respectfully,

Christine Pacinello
Doctoral Candidate
St. John's University
Queens, NY

Appendix E

Mathematics Teaching Self- Efficacy and Instructional Practices Survey*

This survey will ask you about your instructional practices, your classroom strategies, your beliefs and attitudes about teaching and learning mathematics. (*Adapted from the Instructional Interactions Survey, with permission from Dr. Spillane of Northwestern University, School of Education and Social Policy)

Please provide some background information

---

*This survey will be completely anonymous. Names and email addresses will not be collected.

1. Gender
   Mark only one oval.
   - Male
   - Female

2. What grade(s) do you teach this school year? (select all that apply) Check all that apply.
   - Pre- Kindergarten
   - Kindergarten
   - [1]st grade
   - 2nd grade
   - 3rd grade
   - 4th grade
   - 5th grade
   - 6th grade

---

[1] = Strongly disagree, 2= Disagree, 3= Neutral, 4= Agree, 5= Strongly agree.

Please indicate the extent to which you agree or disagree with the following statements:

	1	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

2. I find it difficult to use manipulatives to explain to students why mathematics works. Mark only one oval.

	1	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

3. If students are underachieving in mathematics, it is most likely due to ineffective teaching. Mark only one oval.

	1	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

4. I am typically able to answer students' mathematics questions. Mark only one oval.

	2	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

5. When teaching mathematics, I usually welcome student questions. Mark only one oval.

	3	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

6. I understand mathematics concepts well enough to be an effective elementary mathematics teacher.

Mark only one oval.

	4	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

7. The teacher is generally responsible for the achievement of students in mathematics. Mark only one oval.

	5	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

8. Students' achievement in mathematics is directly related to their teacher's effectiveness in mathematics teaching. Mark only one oval.

	6	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

9. If parents comment that their child is showing more interest in mathematics at school, it is probably due to the performance of the child's teacher. Mark only one oval.

	1	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

10. When the mathematics grades of students improve, it is often due to their teacher having found a more effective teaching approach. Mark only one oval.

	1	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

11. Given a choice, I would not invite the principal to observe my mathematics teaching. Mark only one oval.

	2	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

12. The inadequacy of a student's mathematics background can be overcome by good teaching.
Mark only one oval.

3 2 3 4 5

Strongly disagree ⬭ ⬭ ⬭ ⬭ ⬭ Strongly agree

13. When a student has difficulty understanding a mathematics concept, I am usually at a loss as to how to help the student understand it better. Mark only one oval.

	4	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

14. When a low achieving student shows progress in mathematics, it is usually due to extra attention of the teacher. Mark only one oval.

	5	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

15. I do not know how to get students more interested in mathematics. Mark only one oval.

	1	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

16. When I really try, I can get through to even the most difficult or unmotivated students. Mark only one oval.

	1	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

17. If a student did not remember information I gave in a previous lesson, I would know how to increase his/her retention in the next lesson. Mark only one oval.

	2	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

18. If a student in my class becomes disruptive and noisy, I feel assured that I know some techniques to redirect him/her quickly. Mark only one oval.

	3	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

19. If one of my students couldn't do a class assignment, I would be able to assess accurately whether the assignment was the correct level of difficulty. Mark only one oval.

	4	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

In your mathematics lessons, how often do the students....?

1= Never or almost never 2= some lessons 3= most lessons 4= every lesson

1. Work individually without assistance from the teacher. Mark only one oval.

	1	2	3	4	
Never or almost never	⬭	⬭	⬭	⬭	Every lesson

2. Work individually with assistance from the teacher. Mark only one oval.

	1	2	3	4	
Never or almost never	⬭	⬭	⬭	⬭	Every lesson

3. Work together as a class with the teacher teaching the whole class Mark only one oval.

	1	2	3	4	
Never or almost never	⬭	⬭	⬭	⬭	Every lesson

4. Work together as a class with the students responding to one another. Mark only one oval.

	2	2	3	4	
Never or almost never	⬭	⬭	⬭	⬭	Every lesson

5. Work in pairs or small groups without assistance from each other. Mark only one oval.

	3	2	3	4	
Never or almost never	⬭	⬭	⬭	⬭	Every lesson

6. Work in pairs or small groups with assistance from each other
Mark only one oval.

	4	2	3	4	
Never or almost never	⬭	⬭	⬭	⬭	Every lesson

7. Explain the reasoning behind an idea Mark only one oval.

	5	2	3	4	
Never or almost never	⬭	⬭	⬭	⬭	Every lesson

8. Represent and analyze relationships using tables charts or graphs Mark only one oval.

	6	2	3	4	
Never or almost never	⬭	⬭	⬭	⬭	Every lesson

9. Work on problems for which there are no immediately obvious methods of solution Mark only one oval.

	1	2	3	4	
Never or almost never	⬭	⬭	⬭	⬭	Every lesson

10. Use computers to complete exercises or solve problems Mark only one oval.

	1	2	3	4	
Never or almost never	⬭	⬭	⬭	⬭	Every lesson

11. Write equations to represent relationships Mark only one oval.

	1	2	3	4	
Never or almost never	⬭	⬭	⬭	⬭	Every lesson

Please indicate the extent to which you agree or disagree with the following statements when thinking about the students you currently teach:

1= Strongly disagree, 2= Disagree, 3= Neutral, 4= Agree, 5= Strongly agree.

1. Teachers should encourage students to find their own solutions to math problems even if they are inefficient Mark only one oval.

	1	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

2. Most students have to be shown how to solve simple math problems. Mark only one oval.

	2	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

3. Recall of number facts should precede the development of an understanding of the related operation. Mark only one oval.

3 2 3 4 5

4. Students should master computational procedures before they are expected to understand how those procedures work. Mark only one oval.

	1	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

5. Students need explicit instruction on how to solve word problems. Mark only one oval.

	1	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

6. Teachers should allow students to argue out their own ways to solve simple word problems.

Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

Mark only one oval.

	2	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

7. The goals of instruction in mathematics are best achieved when students find their own methods for solving problems. Mark only one oval.

	3	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

8. Most students can figure out ways to solve many mathematical problems Mark only one oval.

	4	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

9. Time should be spent practicing computational procedures before students are expected to understand the procedures. Mark only one oval.

	5	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

10. Students should not solve simple word problems until they have mastered some number facts.

Mark only one oval.

	1	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

11. Students learn math best by attending to the teacher's explanations. Mark only one oval.

	1	2	3	4	5	
Strongly disagree	⬭	⬭	⬭	⬭	⬭	Strongly agree

12. To be successful in mathematics, a student must be a good listener. Mark only one oval.

2 2 3 4 5

Strongly disagree ⬭ ⬭ ⬭ ⬭ ⬭ Strongly agree

13. Teachers should model specific procedures for solving word problems.
Mark only one oval.

3 2 3 4 5

Strongly disagree ⬭ ⬭ ⬭ ⬭ ⬭ Strongly agree

14. Mathematics should be presented to children in such a way that they can discover relationships for themselves. Mark only one oval.

4 2 3 4 5

Strongly disagree ⬭ ⬭ ⬭ ⬭ ⬭ Strongly agree

15. Students should understand computational procedures before they master them. Mark only one oval.

5 2 3 4 5

Strongly disagree ⬭ ⬭ ⬭ ⬭ ⬭ Strongly agree

16. Time should be spent practicing computational procedures before students spend much time solving problems. Mark only one oval.

6 2 3 4 5

17. Students will not understand an operation until they have mastered some of the relevant number facts. Mark only one oval.

1 2 3 4 5

Strongly disagree ⬭ ⬭ ⬭ ⬭ ⬭ Strongly agree

. 18. Teachers should allow students who are having difficulty solving a word problem to continue to try to find a solution. Mark only one oval.

1 2 3 4 5

Strongly disagree ⬭ ⬭ ⬭ ⬭ ⬭ Strongly agree

Strongly disagree ⬭ ⬭ ⬭ ⬭ ⬭ Strongly agree

Appendix F

Initial coding of survey item using NVIVO11

Survey Item	*Measurement*	*Description*
Section 1		
1	Outcome expectancy	Student Improvement and Teacher effectiveness
2	Self-efficacy beliefs	Teacher welcomes principal observation of mathematics lesson
3	Outcome expectancy	Teacher's ability to overcome student's inadequate math background
4	Self-efficacy beliefs	Teacher's ability to explain concepts
5	Self-efficacy beliefs	Student improvement due to teacher attention
6	Self-efficacy beliefs	Teacher's ability to get students interested in mathematics
7	Outcome expectancy	Teacher Responsibility for Student Achievement
8	Outcome expectancy	Teacher's ability to motivate difficult students
10	Outcome expectancy	Teacher's ability to increase student retention
11	Self-efficacy beliefs	Teacher's ability to redirect difficult students
12	Outcome expectancy	Teacher's ability to respond to students' needs
13	Self-efficacy beliefs	Teacher's ability to explain concepts
14	Outcome expectancy	Student improvement due to teacher attention
15	Self-efficacy beliefs	Respondent's ability to get students interested in mathematics
16	Self-efficacy beliefs	Respondent's ability to motivate difficult students
17	Self-efficacy beliefs	Ability to increase student retention
18	Self-efficacy beliefs	Ability to redirect difficult students
19	Self-efficacy beliefs	Ability to respond to students' needs

Survey Item	*Measurement*	*Description*
Section 2		
1	Instructional Practices	Students work individually without assistance from the teacher
2	Instructional Practices	Students work individually with assistance from the teacher
3	Instructional Practices	Students work together as a class with the teacher teaching the whole class
4	Instructional Practices	Students work together as a class responding to one another
5	Instructional Practices	Students work in small groups without assistance from each other
6	Instructional Practices	Students work in small groups with assistance from each other
7	Instructional Practices	Students explain the reasoning behind an idea
8	Instructional Practices	Students represent and analyze relationships using graphs or tables
9	Instructional Practices	Students work on problems for which there are no immediate methods of solution
10	Instructional Practices	Students use computers to complete exercises or solve problems
11	Instructional Practices	Students write equations to represent relationships

Survey Item	*Measurement*	*Description*
Section 3		
1	Mathematical Beliefs	Encourage students to find their own solutions to math problems even if they are inefficient
2	Mathematical Beliefs	Most students must be shown how to solve simple math problems.
3	Mathematical Beliefs	Recall of number facts should precede the development of an understanding of the related operation.
4	Mathematical Beliefs	Students should master computational procedures before they are expected to understand how those procedures work.
5	Mathematical Beliefs	Students need explicit instruction on how to solve word problems.
6	Mathematical Beliefs	Teachers should allow students to argue out their own ways to solve simple word problems.
7	Mathematical Beliefs	The goals of instruction in mathematics are best achieved when students find their own methods for solving problems.
8	Mathematical Beliefs	Most students can figure out ways to solve many mathematical problems
9	Mathematical Beliefs	Time should be spent practicing computational procedures before students are expected to understand the procedures.
10	Mathematical Beliefs	Students should not solve simple word problems until they have mastered some number facts.
11	Mathematical Beliefs	Students attending to teacher explanations.
12	Mathematical Beliefs	Students must be good listeners
13	Mathematical Beliefs	Teachers should model specific procedures for solving word problems.
14	Mathematical Beliefs	Mathematics should be presented to children in such a way that they can discover relationships for themselves.
15	Mathematical Beliefs	Students should understand computational procedures before they master them.
16	Mathematical Beliefs	Time should be spent practicing computational procedures before students spend much time solving problems.
17	Mathematical Beliefs	Students will not understand an operation until they have mastered some of the relevant number facts.
18	Mathematical Beliefs	Teachers should allow students who are having difficulty solving a word problem to continue to try to find a solution.

Appendix G
Factor Analysis MTSES

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.80	34.55	34.55	3.80	34.55	34.55
2	1.33	12.11	46.67	1.33	12.11	46.67
3	1.09	9.91	56.58	1.09	9.91	56.58
4	0.87	7.87	64.45	0.87	7.87	64.45
5	0.81	7.34	71.78	0.81	7.34	71.78
6	0.68	6.21	78.00	0.68	6.21	78.00
7	0.62	5.65	83.65	0.62	5.65	83.65
8	0.60	5.50	89.15	0.60	5.50	89.15
9	0.48	4.34	93.48			
10	0.43	3.94	97.42			
11	0.28	2.58	100.00			

Pattern Matrix[a]

	Component			
	1	2	3	4
Teacher's ability to use manipulatives to explain mathematics	-.05	-.03	-.02	.95
Teacher's ability to answer student questions	.73	.24	.36	.09
Teacher welcomes student questions	.72	.17	-.04	-.13
Teacher's understanding of mathematical concepts	.79	.04	.04	.05
Teacher welcomes principal observation of mathematics lesson	.61	-.30	-.26	.11
Teacher's ability to explain concepts	.57	-.07	-.52	.11
Teacher's ability to get students interested in mathematics	.28	.10	-.49	.15
Teacher's ability to motivate difficult students	-.15	.30	-.82	.02
Teacher's ability to increase student retention	.02	.72	-.10	-.08
Teacher's ability to redirect difficult students	.03	.56	.02	.39
Teacher's ability to respond to students' needs	.14	.68	-.15	.00

Note. Extraction Method: Principal Component Analysis.
Rotation Method: Oblimin with Kaiser Normalization.[a]

a. Rotation converged in 19 iterations.

Pattern Matrix[a]

	Component		
	1	2	3
Teacher's ability to use manipulatives to explain mathematics	.00	-.01	-.55
Teacher's ability to answer student questions	.80	.23	.10
Teacher welcomes student questions	.60	.19	-.15
Teacher's understanding of mathematical concepts	.69	.05	-.23
Teacher welcomes principal observation of mathematics lesson	.38	-.28	-.56
Teacher's ability to explain concepts	.27	-.02	-.75
Teacher's ability to get students interested in mathematics	.05	.15	-.62
Teacher's ability to motivate difficult students	-.44	.37	-.68
Teacher's ability to increase student retention	.05	.74	.07
Teacher's ability to redirect difficult students	.12	.58	-.11
Teacher's ability to respond to students' needs	.13	.71	-.07

Note. Extraction Method: Principal Component Analysis. Rotation Method: Oblimin with Kaiser Normalization.[a]
a. Rotation converged in 17 iterations.

Pattern Matrix[a]

	Component	
	1	2
Teacher's understanding of mathematical concepts	.81	-.05
Teacher's ability to answer student questions	.72	-.06
Teacher welcomes principal observation of mathematics lesson	.72	-.12
Teacher's ability to explain concepts	.67	.24
Teacher welcomes student questions	.66	.07
Teacher's ability to use manipulatives to explain mathematics	.29	.24
Teacher's ability to motivate difficult students	-.12	.78
Teacher's ability to respond to students' needs	.08	.67
Teacher's ability to increase student retention	-.08	.66
Teacher's ability to redirect difficult students	.11	.57
Teacher's ability to get students interested in mathematics	.36	.41

Note. Extraction Method: Principal Component Analysis.
Rotation Method: Oblimin with Kaiser Normalization.[a]

a. Rotation converged in 6 iterations.

Component Correlation Matrix

Component	1	2
1	1	0.39
2	.39	1.00

Note. Extraction Method: Principal Component Analysis.
Rotation Method: Oblimin with Kaiser Normalization.

Appendix H
Jury Instrument: Components of Mathematics Teaching Self-Efficacy Score

Directions:
The following survey questions relate to Mathematics Self- Efficacy. Please indicate with an X where each item best fits.

Survey Item	Management/ Classroom Climate	Communication Clarification and Feedback	Accommodatin g individual differences	Motivation of students
#2. I find it difficult to use manipulatives to explain why mathematics work.				
#4. I am typically able to answer students' mathematics questions				
#5. When teaching mathematics, I usually welcome student questions.				
#6. I understand mathematics concepts well enough to be an effective elementary mathematics teacher.				

Survey Item	Management/ Classroom Climate	Communication/Clarification and Feedback	Accommodating individual differences	Motivation Of students
#11. Given a choice, I would not invite the principal to observe my mathematics teaching.				
#13. When a student has difficulty understanding a mathematics concept, I am usually at a loss as to how to help a student understand it better.				
#15 I do not know how to get students more interested in mathematics.				
#16 When I really try, I can get through to even the most difficult or unmotivated students.				
#17. If a student did not remember information I gave in a previous lesson, I would know how to increase retention.				

Survey Item	Managemen t/ Classroom Climate	Communicati on/Clarificatio n and Feedback	Accommodati ng individual differences	Motivation Of students
#18. If a student in my class becomes noisy and disruptive, I feel assured that I know some techniques to redirect him/her.				
#19 If one of my students couldn’t do a class assignment, I would be able to assess accurately whether the assignment was the correct level of difficulty.				

Appendix I
Factor Analysis MIP_Adj

	Initial Eigenvalues	Extraction Sums of Squared Loadings		
Component	Total	Total	% of Variance	Cumulative %
1	2.66	2.66	24.20	24.20
2	1.78	1.78	16.17	40.37
3	1.31	1.31	11.87	52.24
4	0.98			
5	0.90			
6	0.76			
7	0.72			
8	0.60			
9	0.56			
10	0.48			
11	0.25			

Pattern Matrix

	Component		
	1	2	3
1. Work individually without assistance from the teacher.	.38	.48	-.01
2. Work individually with assistance from the teacher.	.00	.68	.00
3. Work together as a class with the teacher teaching the whole class	-.12	.86	-.10
4. Work together as a class with the students responding to one another.	.15	.53	.19
5. Work in pairs or small groups without assistance from each other.	-.40	.19	.82
6. Work in pairs or small groups with assistance from each other	.00	-.10	.74
7. Explain the reasoning behind an idea	.59	.13	-.07
8. Represent and analyze relationships using tables charts or graphs	.46	-.26	.52
9. Work on problems for which there are no immediately obvious methods of solution	.31	.11	.46
10. Use computers to complete exercises or solve problems	.48	.04	.38
11. Write equations to represent relationships	.82	.01	-.11

Extraction Method: Principal Component Analysis.
Rotation Method: Oblimin with Kaiser Normalization.

a. Rotation converged in 23 iterations.

Appendix J

Principle Components Analysis of Mathematical Beliefs Inventory

Total Variance Explained

Component	Total	Total	% of Variance	Cumulative %	Total
1	3.35	3.35	18.63	18.63	2.94
2	2.37	2.37	13.15	31.78	2.68
3	1.85	1.85	10.28	42.06	2.31
4	1.35				
5	1.22				
6	1.13				
7	1.07				
8	0.92				
9	0.79				
10	0.76				
11	0.70				
12	0.54				
13	0.50				
14	0.45				
15	0.34				
16	0.26				
17	0.23				
18	0.18				

Note. Extraction Method: Principal Component Analysis.

a. When components are correlated, sums of squared loadings cannot be added to obtain a total variance.

Pattern Matrix[a]

	Component		
	1	2	3
Encourage students to find their own solutions to math problems even if they are inefficient	.20	.30	-.08
Most students have to be shown how to solve simple math problems.	.32	.02	-.07
Recall of number facts should precede the development of an understanding of the related operation.	.72	-.04	.06
Students should master computational procedures before they are expected to understand how those procedures work.	.82	.02	.31
Students need explicit instruction on how to solve word problems.	.05	.26	-.33
Teachers should allow students to argue out their own ways to solve simple word problems.	-.16	.70	.10
The goals of instruction in mathematics are best achieved when students find their own methods for solving problems.	-.08	.83	.19
Most students can figure out ways to solve many mathematical problems	.22	.55	.27
Time should be spent practicing computational procedures before students are expected to understand the procedures.	.72	-.11	-.06
Students should not solve simple word problems until they have mastered some number facts.	.08	.52	-.19
Students attending to teacher explanations	.14	.38	-.29
Students must be good listeners	-.04	.39	-.09
Teachers should model specific procedures for solving word problems.	-.13	.33	-.67
Mathematics should be presented to children in such a way that they can discover relationships for themselves.	.10	.33	.64
Students should understand computational procedures before they master them.	-.14	.17	.74
Time should be spent practicing computational procedures before students spend much time solving problems.	.47	.02	-.57
Students will not understand an operation until they have mastered some of the relevant number facts.	.72	.05	-.08
Teachers should allow students who are having difficulty solving a word problem to continue to try to find a solution.	-.01	.33	.03

Extraction Method: Principal Component Analysis.
Rotation Method: Oblimin with Kaiser Normalization.[a]

Appendix K
Jury Instrument: Components of Instructional Practices
Directions:
The following survey questions relate to Mathematics Instructional Practices. Please indicate with an X where each item best fits.

Survey Item "In your classroom, how often do students…."	Discourse (Discussion or Conversation)	Inquiry (Curiosity, Challenge and Connection Making)	Collaboration	Creativity and Higher order thinking skills
P1. Work individually without assistance from the teacher.				
P2. Work individually with assistance from the teacher.				
P3. Work together as a class with the teacher teaching the whole class				
P4. Work together as a class with the students responding to one another.				
P5. Work in pairs or small groups without assistance from each other..				
P6. Work in pairs or small groups with assistance from each other				

Survey Item "In your classroom, how often do students…."	Discourse (Discussion or Conversation)	Inquiry (Curiosity, Challenge and Connection Making)	Collaboration	Creativity and Higher order thinking skills
P7. Explain the reasoning behind an idea				
P8. Represent and analyze relationships using tables charts or graphs				
P9. Work on problems for which there are no immediately obvious methods of solution				
P10. Use computers to complete exercises or solve problems				
P11. Write equations to represent relationships				